openstax™ COLLEGE

Student Solutions Manual

College

Physics

CONTENTS

PREFACE

The Student's Solutions Manual provides solutions to select Problems & Exercises from Openstax *College Physics*. The purpose of this manual and of the Problems & Exercises is to build problem-solving skills that are critical to understanding and applying the principles of physics. The text of *College Physics* contains many features that will help you not only to solve problems, but to understand their concepts, including Problem-Solving Strategies, Examples, Section Summaries, and chapter Glossaries. Before turning to the problem solutions in this manual, you should use these features in your text to your advantage. The worst thing you can do with the solutions manual is to copy the answers directly without thinking about the problem-solving process and the concepts involved.

The text of *College Physics* is available in multiple formats (online, PDF, e-pub, and print) from http://openstaxcollege.org/textbooks/college-physics. While these multiple formats provide you with a wide range of options for accessing and repurposing the text, they also present some challenges for the organization of this solutions manual, since problem numbering is automated and the same problem may be numbered differently depending on the format selected by the end user.

As such, we have decided to organize the Problems & Exercises manual by chapter and section, as they are organized in the PDF and print versions of *College Physics.* See the Table of Contents on the previous page.

Problem numbering throughout the solutions manual will match the numbering in the PDF version of the product, provided that users have not modified or customized the original content of the book by adding or removing problems. Numbering of Tables, Figures, Examples, and other elements of the text throughout this manual will also coincide with the numbering in the PDF and print versions of the text.

For online and epub users of *College Physics,* we have included question stem along with the solution for each Problem order to minimize any confusion caused by discrepancies in numbering. Images, figures, and tables—which occasionally accompany or complement problems and exercises—have been omitted from the solutions manual to save space.

CHAPTER 1: INTRODUCTION: THE NATURE OF SCIENCE AND PHYSICS

1.2 PHYSICAL QUANTITIES AND UNITS

4. *American football is played on a 100-yd-long field, excluding the end zones. How long is the field in meters? (Assume that 1 meter equals 3.281 feet.)*

Solution Since 3 feet = 1 yard and 3.281 feet = 1 meter, multiply 100 yards by these conversion factors to cancel the units of yards, leaving the units of meters:

$$100\text{ yd} = 100\text{ yd} \times \frac{3\text{ ft}}{1\text{ yd}} \times \frac{1\text{ m}}{3.281\text{ ft}} = \underline{91.4\text{ m}}$$

A football field is 91.4 m long.

10. *(a) Refer to Table 1.3 to determine the average distance between the Earth and the Sun. Then calculate the average speed of the Earth in its orbit in kilometers per second. (b) What is this in meters per second?*

Solution (a) The average speed of the earth's orbit around the sun is calculated by dividing the distance traveled by the time it takes to go one revolution:

$$\text{average speed} = \frac{2\pi(\text{average dist of Earth to sun})}{1\text{ year}}$$

$$= \frac{2\pi(10^8\text{ km})}{365.25\text{ d}} \times \frac{1\text{ d}}{24\text{ h}} \times \frac{1\text{ h}}{3600\text{ s}} = \underline{20\text{ km/s}}$$

The earth travels at an average speed of 20 km/s around the sun.

(b) To convert the average speed into units of m/s, use the conversion factor: 1000 m

= 1 km:

$$\text{average speed} = \frac{20 \text{ km}}{\text{s}} \times \frac{1000 \text{ m}}{1 \text{ km}} = \underline{20 \times 10^3 \text{ m/s}}$$

1.3 ACCURACY, PRECISION, AND SIGNIFICANT FIGURES

15. *(a) Suppose that a person has an average heart rate of 72.0 beats/min. How many beats does he or she have in 2.0 y? (b) In 2.00 y? (c) In 2.000 y?*

Solution (a) To calculate the number of beats she has in 2.0 years, we need to multiply 72.0 beats/minute by 2.0 years and use conversion factors to cancel the units of time:

$$\frac{72.0 \text{ beats}}{1 \text{ min}} \times \frac{60.0 \text{ min}}{1.00 \text{ h}} \times \frac{24.0 \text{ h}}{1.00 \text{ d}} \times \frac{365.25 \text{ d}}{1.00 \text{ y}} \times 2.0 \text{ y} = 7.5738 \times 10^7 \text{ beats}$$

Since there are only 2 significant figures in 2.0 years, we must report the answer with 2 significant figures: $\underline{7.6 \times 10^7 \text{ beats}}$.

(b) Since we now have 3 significant figures in 2.00 years, we now report the answer with 3 significant figures: $\underline{7.57 \times 10^7 \text{ beats}}$.

(c) Even though we now have 4 significant figures in 2.000 years, the 72.0 beats/minute only has 3 significant figures, so we must report the answer with 3 significant figures: $\underline{7.57 \times 10^7 \text{ beats}}$.

21. *A person measures his or her heart rate by counting the number of beats in* 30 s*. If* 40 ± 1 *beats are counted in* $30.0 \pm 0.5 \text{ s}$*, what is the heart rate and its uncertainty in beats per minute?*

Solution To calculate the heart rate, we need to divide the number of beats by the time and convert to beats per minute.

$$\frac{\text{beats}}{\text{minute}} = \frac{40\ \text{beats}}{30.0\ \text{s}} \times \frac{60.0\ \ \text{s}}{1.00\ \text{min}} = 80\ \text{beats/min}$$

To calculate the uncertainty, we use the method of adding percents.

$$\%\,unc = \frac{1\ \text{beat}}{40\ \text{beats}} \times 100\% + \frac{0.5\ \text{s}}{30.0\ \text{s}} \times 100\% = 2.5\% + 1.7\% = 4.2\% = 4\%$$

Then calculating the uncertainty in beats per minute:

$$\delta\ A = \frac{\%\,unc}{100\%} \times A = \frac{4.2\%}{100\%} \times 80\ \text{beats/min} = 3.3\ \text{beats/min} = 3\ \text{beats/min}$$

Notice that while doing calculations, we keep one EXTRA digit, and round to the correct number of significant figures only at the end.

So, the heart rate is $\underline{80 \pm 3\ \text{beats/min}}$.

27. *The length and width of a rectangular room are measured to be* $3.955 \pm 0.005\ \text{m}$ *and* $3.050 \pm 0.005\ \text{m}$. *Calculate the area of the room and its uncertainty in square meters.*

Solution The area is $3.995\ \text{m} \times 3.050\ \text{m} = 12.06\ \text{m}^2$. Now use the method of adding percents to get uncertainty in the area.

$$\%\,unc\ \text{length} = \frac{0.005\ \text{m}}{3.955\ \text{m}} \times 100\% = 0.13\%$$

$$\%\,unc\ \text{width} = \frac{0.005\ \text{m}}{3.050\ \text{m}} \times 100\% = 0.16\%$$

$$\%\,unc\ \text{area} = \%\,unc\ \text{length} + \%\,unc\ \text{width} = 0.13\% + 0.16\% = 0.29\% = 0.3\%$$

Finally, using the percent uncertainty for the area, we can calculate the uncertainty in square meters: $\delta\ area = \frac{\%\,unc\ \text{area}}{100\%} \times \text{area} = \frac{0.29\%}{100\%} \times 12.06\ \text{m}^2 = 0.035\ \text{m}^2 = 0.04\ \text{m}^2$

The area is $\underline{12.06\ \pm\ 0.04\ \ \text{m}^2}$.

CHAPTER 2: KINEMATICS

2.1 DISPLACEMENT

1. *Find the following for path A in Figure 2.59: (a) The distance traveled. (b) The magnitude of the displacement from start to finish. (c) The displacement from start to finish.*

Solution (a) The total distance traveled is the length of the line from the dot to the arrow in path A, or $\underline{7\text{ m}}$.

(b) The distance from start to finish is the magnitude of the difference between the position of the arrows and the position of the dot in path A:

$$\Delta x = |x_2 - x_1| = |7\text{ m} - 0\text{ m}| = \underline{7\text{ m}}$$

(c) The displacement is the difference between the value of the position of the arrow and the value of the position of the dot in path A: The displacement can be either positive or negative: $\Delta x = x_2 - x_1 = 7\text{ m} - 0\text{ m} = \underline{+7\text{ m}}$

2.3 TIME, VELOCITY, AND SPEED

14. *A football quarterback runs 15.0 m straight down the playing field in 2.50 s. He is then hit and pushed 3.00 m straight backward in 1.75 s. He breaks the tackle and runs straight forward another 21.0 m in 5.20 s. Calculate his average velocity (a) for each of the three intervals and (b) for the entire motion.*

Solution (a) The average velocity for the first segment is the distance traveled downfield (the positive direction) divided by the time he traveled:

$$\bar{v}_1 = \frac{\text{displacement}}{\text{time}} = \frac{+15.0\text{ m}}{2.50\text{ s}} = \underline{+6.00\text{ m/s (forward)}}$$

The average velocity for the second segment is the distance traveled (this time in the negative direction because he is traveling backward) divided by the time he traveled: $\bar{v}_2 = \dfrac{-3.00\text{ m}}{1.75\text{ s}} = \underline{-1.71\text{ m/s (backward)}}$

Finally, the average velocity for the third segment is the distance traveled (positive again because he is again traveling downfield) divided by the time he traveled: $\bar{v}_3 = \dfrac{+21.0\text{ m}}{5.20\text{ s}} = \underline{+4.04\text{ m/s(forward)}}$

(b) To calculate the average velocity for the entire motion, we add the displacement from each of the three segments (remembering the sign of the numbers), and divide by the total time for the motion:

$$\bar{v}_{\text{total}} = \frac{15.0\text{ m} - 3.00\text{ m} + 21.0\text{ m}}{2.50\text{ s} + 1.75\text{ s} + 5.20\text{ s}} = \underline{+3.49\text{ m/s}}$$

Notice that the average velocity for the entire motion is not just the addition of the average velocities for the segments.

15. *The planetary model of the atom pictures electrons orbiting the atomic nucleus much as planets orbit the Sun. In this model you can view hydrogen, the simplest atom, as having a single electron in a circular orbit* $1.06 \times 10^{-10}\text{ m}$ *in diameter. (a) If the average speed of the electron in this orbit is known to be* $2.20 \times 10^{6}\text{ m/s}$*, calculate the number of revolutions per second it makes about the nucleus. (b) What is the electron's average velocity?*

Solution (a) The average speed is defined as the total distance traveled divided by the elapsed time, so that: $\text{average speed} = \dfrac{\text{distance traveled}}{\text{time elapsed}} = 2.20 \times 10^{6}\text{ m/s}$

If we want to find the number of revolutions per second, we need to know how far the electron travels in one revolution.

$$\frac{\text{distance traveled}}{\text{revolution}} = \frac{2\pi r}{1\text{ rev}} = \frac{2\pi[(0.5)(1.06 \times 10^{-10}\text{ m})]}{1\text{ rev}} = \frac{3.33 \times 10^{-10}\text{ m}}{1\text{ rev}}$$

So to calculate the number of revolutions per second, we need to divide the average speed by the distance traveled per revolution, thus canceling the units of meters: $\frac{\text{rev}}{\text{s}} = \frac{\text{average speed}}{\text{distance/revolution}} = \frac{2.20\times10^6 \text{ m/s}}{3.33\times10^{-10}\text{ m/revolution}} = \underline{6.61\times10^{15}\text{ rev/s}}$

(b) The velocity is defined to be the displacement divided by the time of travel, so since there is no *net* displacement during any one revolution: $\underline{v = 0\text{ m/s}}$.

2.5 MOTION EQUATIONS FOR CONSTANT ACCELERATION IN ONE DIMENSION

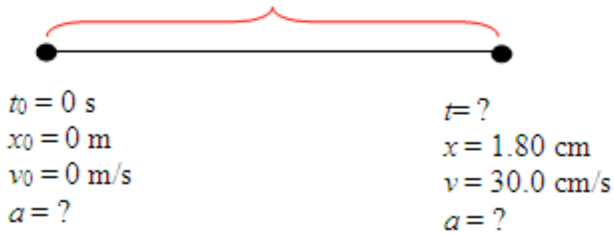
21. *A well-thrown ball is caught in a well-padded mitt. If the deceleration of the ball is* $2.10\times10^4 \text{ m/s}^2$*, and 1.85 ms* $(1\text{ ms} = 10^{-3}\text{ s})$ *elapses from the time the ball first touches the mitt until it stops, what was the initial velocity of the ball?*

Solution Given: $a = -2.10\times10^4 \text{ m/s}^2; t = 1.85\text{ ms} = 1.85\times10^{-3}\text{s}; v = 0\text{ m/s}$, find v_0. We use the equation $v_0 = v - at$ because it involves only terms we know and terms we want to know. Solving for our unknown gives:

$v_0 = v - at = 0\text{ m/s} - (-2.10\times10^4 \text{ m/s}^2)(1.85\times10^{-3}\text{ s}) = \underline{38.9\text{ m/s}}$ (about 87 miles per hour)

26. ***Professional Application*** *Blood is accelerated from rest to 30.0 cm/s in a distance of 1.80 cm by the left ventricle of the heart. (a) Make a sketch of the situation. (b) List the knowns in this problem. (c) How long does the acceleration take? To solve this part, first identify the unknown, and then discuss how you chose the appropriate equation to solve for it. After choosing the equation, show your steps in solving for the unknown, checking your units. (d) Is the answer reasonable when compared with the time for a heartbeat?*

Solution (a)



(b) Knowns: “Accelerated from rest” $\Rightarrow$ $v_0 = 0\ \text{m/s}$

“to 30.0 cm/s” $\Rightarrow$ $v = 0.300\ \text{m/s}$
“in a distance of 1.80 cm” $\Rightarrow$ $x - x_0 = \underline{0.0180\ \text{m}}$.

(c) “How long” tells us to find t. To determine which equation to use, we look for an equation that has $v_0, v, x - x_0$ and t, since those are parameters that we know or want to know. Using the equations $x = x_0 + \bar{v}t$ and $\bar{v} = \dfrac{v_0 + v}{2}$ gives

$$x - x_0 = \left(\frac{v_0 + v}{2}\right)t.$$

Solving for t gives: $t = \dfrac{2(x - x_0)}{v_0 + v} = \dfrac{2(0.0180\ \text{m})}{(0\ \text{m/s}) + (0.300\ \text{m/s})} = \underline{0.120\ \text{s}}$

It takes 120 ms to accelerate the blood from rest to 30.0 cm/s. Converting everything to standard units first makes it easy to see that the units of meters cancel, leaving only the units of seconds.

(d) Yes, the answer is reasonable. An entire heartbeat cycle takes about one second. The time for acceleration of blood out of the ventricle is only a fraction of the entire cycle.

32. ***Professional Application*** *A woodpecker's brain is specially protected from large decelerations by tendon-like attachments inside the skull. While pecking on a tree, the woodpecker's head comes to a stop from an initial velocity of 0.600 m/s in a distance of only 2.00 mm. (a) Find the acceleration in* m/s^2 *and in multiples of* $g\left(g = 9.80\ \text{m/s}^2\right)$. *(b) Calculate the stopping time. (c) The tendons cradling the brain stretch, making its stopping distance 4.50 mm (greater than the head and, hence, less deceleration of the brain). What is the brain's deceleration, expressed in multiples of* g*?*

Solution (a) Find a (which should be negative).

Given: "comes to a stop" $\Rightarrow v = 0\ \text{m/s}$.

"from an initial velocity of" $\Rightarrow v_0 = 0.600\ \text{m/s}$.

"in a distance of 2.00 m" $\Rightarrow x - x_0 = 2.00 \times 10^{-3}\ \text{m}$.

So, we need an equation that involves a, v, v_0, and $x - x_0$, or the equation

$v^2 = v_0{}^2 + 2a(x - x_0)$, so that

$$a = \frac{v^2 - v_0^2}{2(x - x_0)} = \frac{(0\ \text{m/s})^2 - (0.600\ \text{m/s})^2}{2(2.00 \times 10^{-3}\ \text{m})} = \underline{-90.0\ \text{m/s}^2}$$

So the deceleration is $90.0\ \text{m/s}^2$. To get the deceleration in multiples of g, we divide a by g: $\dfrac{|a|}{|g|} = \dfrac{90.0\ \text{m/s}^2}{9.80\ \text{m/s}^2} = 9.18 \Rightarrow a = \underline{9.18g}.$

(b) The words "Calculate the stopping time" mean find t. Using $x - x_0 = \frac{1}{2}(v_0 + v)t$

gives $x - x_0 = \frac{1}{2}(v_0 + v)t$, so that

$$t = \frac{2(x - x_0)}{v_0 + v} = \frac{2(2.00 \times 10^{-3}\ \text{m})}{(0.600\ \text{m/s}) + (0\ \text{m/s})} = \underline{6.67 \times 10^{-3}\ \text{s}}$$

(c) To calculate the deceleration of the brain, use $x - x_0 = 4.50\ \text{mm} = 4.50 \times 10^{-3}\ \text{m}$

instead of 2.00 mm. Again, we use $a=\frac{v^2-v_0^2}{2(x-x_0)}$, so that:

$$a=\frac{v^2-v_0^2}{2(x-x_0)}=\frac{(0\text{ m/s})^2-(0.600\text{ m/s})^2}{2(4.50\times10^{-3}\text{ m})}=\underline{-40.0\text{ m/s}^2}$$

And expressed in multiples of *g* gives: $\frac{|a|}{|g|}=\frac{40.0\text{ m/s}^2}{9.80\text{ m/s}^2}=4.08\Rightarrow a=4.08g$

2.7 FALLING OBJECTS

41. *Calculate the displacement and velocity at times of (a) 0.500, (b) 1.00, (c) 1.50, and (d) 2.00 s for a ball thrown straight up with an initial velocity of 15.0 m/s. Take the point of release to be* $y_0=0$.

Solution

Knowns: $a=\text{acceleration due to gravity}=g=-9.8\text{ m/s}^2;\ y_0=0\text{ m};\ v_0=+15.0\text{ m/s}$

To find displacement we use $y=y_0+v_0t+\frac{1}{2}at^2$, and to find velocity we use $v=v_0+at$.

(a) $y_1=y_0+v_0t_1+\frac{1}{2}at_1^2$

$$=0\text{ m}+(15.0\text{ m/s})(0.500\text{ s})+\frac{1}{2}(-9.8\text{ m/s}^2)(0.500\text{ s})^2=\underline{6.28\text{ m}}$$

$$v_1=v_0+at_1=(15.0\text{ m/s})+(-9.8\text{ m/s}^2)(0.500\text{ s})=\underline{10.1\text{ m/s}}$$

(b) $y_2=y_0+v_0t_2+\frac{1}{2}at_2^2$

$$=0\text{ m}+(15.0\text{ m/s})(1.00\text{ s})+\frac{1}{2}(-9.8\text{ m/s}^2)(1.00\text{ s})^2=\underline{10.1\text{ m}}$$

$$v_2=v_0+at_2=(15.0\text{ m/s})+(-9.8\text{ m/s}^2)(1.00\text{ s})=\underline{5.20\text{ m/s}}$$

(c) $y_3 = y_0 + v_0 t_3 + \frac{1}{2} a t_3^2$

$= 0\text{ m} + (15.0\text{ m/s})(1.50\text{ s}) + \frac{1}{2}(-9.8\text{ m/s}^2)(1.50\text{ s})^2 = \underline{11.5\text{ m}}$

$v_3 = v_0 + a t_3 = (15.0\text{ m/s}) + (-9.8\text{ m/s}^2)(1.50\text{ s}) = \underline{0.300\text{ m/s}}$

The ball is almost at the top.

(d) $y_4 = y_0 + v_0 t_4 + \frac{1}{2} a t_4^2$

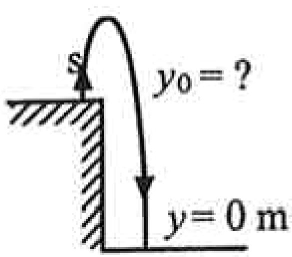
$= 0\text{ m} + (15.0\text{ m/s})(2.00\text{ s}) + \frac{1}{2}(-9.8\text{ m/s}^2)(2.00\text{ s})^2 = \underline{10.4\text{ m}}$

$v_4 = v_0 + a t_4 = (15.0\text{ m/s}) + (-9.8\text{ m/s}^2)(2.00\text{ s}) = \underline{-4.60\text{ m/s}}$

The ball has begun to drop.

47. *(a) Calculate the height of a cliff if it takes 2.35 s for a rock to hit the ground when it is thrown straight up from the cliff with an initial velocity of 8.00 m/s. (b) How long would it take to reach the ground if it is thrown straight down with the same speed?*

Solution



(a) Knowns: $t = 2.35\text{ s}; y = 0\text{ m}; v_0 = +8.00\text{ m/s}; a = -9.8\text{ m/s}^2$

Since we know t, y, v_0, and a and want to find y_0, we can use the equation $y = y_0 + v_0 t + \frac{1}{2} a t^2$.

$y = (0\text{ m}) + (+8.00\text{ m/s})(2.35\text{ s}) + \frac{1}{2}(-9.80\text{ m/s}^2)(2.35\text{ s})^2 = -8.26\text{ m}$, so the cliff is <u>8.26 m</u> high.

(b) Knowns: $y = 0\text{ m}; y_0 = 8.26\text{ m}; v_0 = -8.00\text{ m/s}; a = -9.80\text{ m/s}^2$

Now we know y, y_0, v_0, and a and want to find t, so we use the equation $y = y_0 + v_0 t + \frac{1}{2}at^2$ again. Rearranging,

$$t = \frac{-v_0 \pm \sqrt{v_0^2 - 4(0.5a)(y_0 - y)}}{2(0.5a)}$$

$$t = \frac{-(-8.00\text{ m/s}) \pm \sqrt{(-8.00\text{ m/s})^2 + 2(9.80\text{ m/s}^2)(8.26\text{ m} - 0\text{ m})}}{(-9.80\text{ m/s}^2)}$$

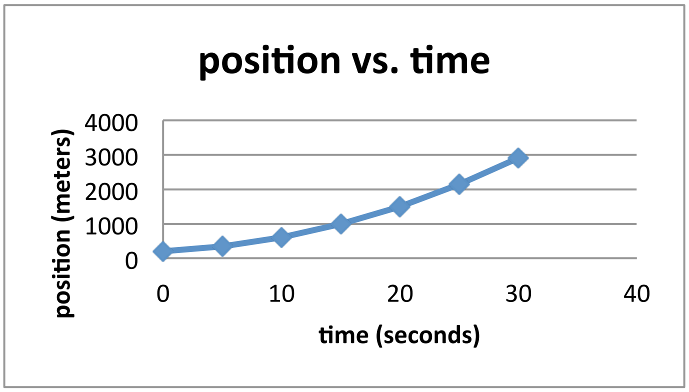
$$= \frac{8.00\text{ m/s} \pm 15.03\text{ m/s}}{-9.80\text{ m/s}^2}$$

$$t = 0.717\text{ s or } -2.35\text{ s} \Rightarrow t = \underline{0.717\text{ s}}$$

2.8 GRAPHICAL ANALYSIS OF ONE-DIMENSIONAL MOTION

59. *(a) By taking the slope of the curve in Figure 2.60, verify that the velocity of the jet car is 115 m/s at* $t = 20\text{ s}$*. (b) By taking the slope of the curve at any point in Figure 2.61, verify that the jet car's acceleration is* 5.0 m/s^2*.*

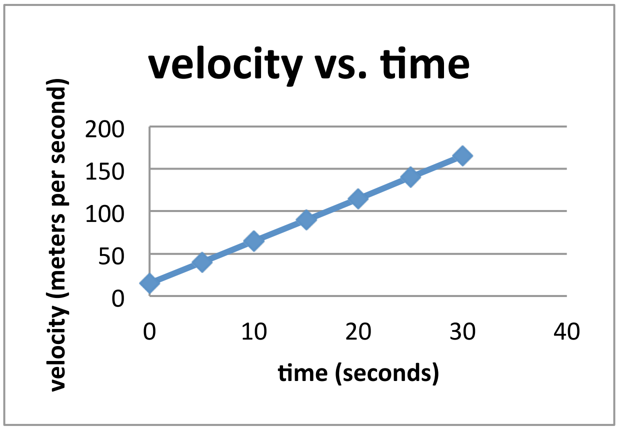
Solution (a)



In the position vs. time graph, if we draw a tangent to the curve at $t = 20\text{ s}$, we can identify two points: $x = 0\text{ m}, t = 5\text{ s}$ and $x = 1500\text{ m}, t = 20\text{ s}$ so we can calculate an approximate slope: $v = \frac{\text{rise}}{\text{run}} = \frac{(2138 - 988)\text{ m}}{(25 - 15)\text{ s}} = 115\text{ m/s}$

So, the slope of the displacement vs. time curve is the velocity curve.

(b)



In the velocity vs. time graph, we can identify two points: $v = 65\ \text{m/s}, t = 10\ \text{s}$ and $v = 140\ \text{m/s}, t = 25\ \text{s}$. Therefore , the slope is $a = \dfrac{\text{rise}}{\text{run}} = \dfrac{(140 - 65)\ \text{m/s}}{(25 - 10)\ \text{s}} = 5.0\ \text{m/s}^2$

The slope of the velocity vs. time curve is the acceleration curve.

CHAPTER 3: TWO-DIMENSIONAL KINEMATICS

3.2 VECTOR ADDITION AND SUBTRACTION: GRAPHICAL METHODS

1. *Find the following for path A in Figure 3.54: (a) the total distance traveled, and (b) the magnitude and direction of the displacement from start to finish.*

Solution (a) To measure the total distance traveled, we take a ruler and measure the length of Path A to the north, and add to it to the length of Path A to the east. Path A travels 3 blocks north and 1 block east, for a total of four blocks. Each block is 120 m, so the distance traveled is $d = (4 \times 120\ \text{m}) = \underline{480\ \text{m}}$

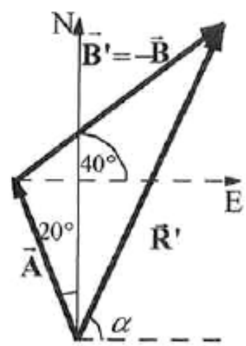
(b) Graphically, measure the length and angle of the line from the start to the arrow of Path A. Use a protractor to measure the angle, with the center of the protractor at the start, measure the angle to where the arrow is at the end of Path A. In order to do this, it may be necessary to extend the line from the start to the arrow of Path A, using a ruler. The length of the displacement vector, measured from the start to the arrow of Path A, along the line you just drew.

$\underline{\mathbf{S} = 379\ \text{m}, 18.4^\circ\ \text{E of N}}$

7. *Repeat the problem two problems prior, but for the second leg you walk 20.0 m in a direction* 40.0° *north of east (which is equivalent to subtracting* $\mathbf{B}$ *from* $\mathbf{A}$ *—that is, to finding* $\mathbf{R'} = \mathbf{A} - \mathbf{B}$*). (b) Repeat the problem two problems prior, but now you first walk 20.0 m in a direction* 40.0° *south of west and then 12.0 m in a direction* 20.0° *east of south (which is equivalent to subtracting* $\mathbf{A}$ *from* $\mathbf{B}$ *—that is, to finding* $\mathbf{R''} = \mathbf{B} - \mathbf{A} = -\mathbf{R'}$*). Show that this is the case.*

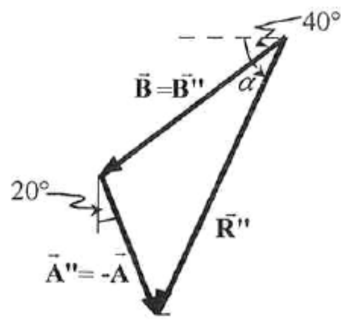
Solution (a) To do this problem, draw the two vectors **A** and **B’** = **–B** tip to tail as shown below. The vector **A** should be 12.0 units long and at an angle of $20°$ to the left of the *y*-axis. Then at the arrow of vector **A**, draw the vector **B’** = **–B**, which should be 20.0 units long and at an angle of $40°$ above the *x*-axis. The resultant vector, **R’**, goes from the tail of vector **A** to the tip of vector **B**, and therefore has an angle of α above the *x*-axis. Measure the length of the resultant vector using your ruler, and use a protractor with center at the tail of the resultant vector to get the angle.

$\underline{R' = 26.6\ \text{m, and}\ \alpha = 65.1°\ \text{N of E}}$



(b) To do this problem, draw the two vectors **B** and **A”** = **–A** tip to tail as shown below. The vector **B** should be 20.0 units long and at an angle of $40°$ below the *x*-axis. Then at the arrow of vector **B**, draw the vector **A”** = **–A**, which should be 12.0 units long and at an angle of $20°$ to the right of the negative *y*-axis. The resultant vector, **R”**, goes from the tail of vector **B** to the tip of vector **A”**, and therefore has an angle of α below the *x*-axis. Measure the length of the resultant vector using your ruler, and use a protractor with center at the tail of the resultant vector to get the angle.

$\underline{R'' = 26.6\ \text{m, and}\ \alpha = 65.1°\ \text{S of W}}$



So the length is the same, but the direction is reversed from part (a).

3.3 VECTOR ADDITION AND SUBTRACTION: ANALYTICAL METHODS

13. *Find the following for path C in Figure 3.58: (a) the total distance traveled and (b) the magnitude and direction of the displacement from start to finish. In this part of the problem, explicitly show how you follow the steps of the analytical method of vector addition.*

Solution (a) To solve this problem analytically, add up the distance by counting the blocks traveled along the line that is Path C:

$$d = (1\times 120\text{ m}) + (5\times 120\text{ m}) + (2\times 120\text{ m}) + (1\times 120\text{ m}) + (1\times 120\text{ m}) + (3\times 120\text{ m})$$
$$= \underline{1.56\times 10^3\text{ m}}$$

(b) To get the displacement, calculate the displacements in the *x*- and *y*- directions separately, then use the formulas for adding vectors. The displacement in the *x*-direction is calculated by adding the *x*-distance traveled in each leg, being careful to subtract values when they are negative:

$$s_x = (0 + 600 + 0 - 120 + 0 - 360)\text{m} = 120\text{ m}$$

Using the same method, calculate the displacement in the *y*-direction:

$$s_y = (120 + 0 - 240 + 0 + 120 + 0)\text{m} = 0\text{ m}$$

Now using the equations $R = \sqrt{R_x^2 + R_y^2}$ and $\theta = \tan^{-1}\left(R_x / R_y \right)$, calculate the total displacement vectors:

$$s = \sqrt{s^2{}_x + s^2{}_y} = \sqrt{(120\text{ m})^2 + (0\text{ m})^2} = 120\text{ m}$$
$$\theta = \tan^{-1}\left(S_y / S_x \right) = \tan^{-1}\left(0\text{ m} / 120\text{ m}\right)$$

$0° \Rightarrow \text{east}$, so that <u>**S** = 120 m, east</u>

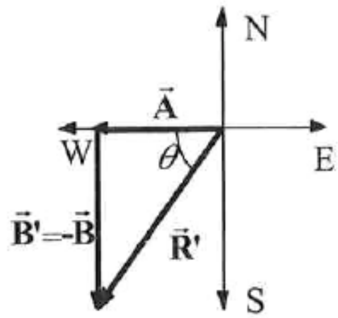
19. *Do Problem 3.16 again using analytical techniques and change the second leg of the walk to 25.0 m straight south. (This is equivalent to subtracting $\mathbf{B}$ from $\mathbf{A}$—that is, finding $\mathbf{R'} = \mathbf{A} - \mathbf{B}$) (b) Repeat again, but now you first walk 25.0 m north and then 18.0 m east. (This is equivalent to subtract $\mathbf{A}$ from $\mathbf{B}$—that is, to find $\mathbf{A} = \mathbf{B} + \mathbf{C}$. Is that consistent with your result?)*

Solution (a) We want to calculate the displacement for walk 18.0 m to the west, followed by 25.0 m to the south. First, calculate the displacement in the *x*- and *y*-directions, using the equations $R_x = A_x + B_x$ and $R_y = A_y + B_y$: (the angles are measured from due east).

$$R_x = -18.0\text{ m}, R_y = -25.0\text{ m}$$

Then, using the equations $R = \sqrt{R_x^2 + R_y^2}$ and $\theta = \tan^{-1}\left(R_x / R_y \right)$, calculate the total displacement vectors:

$$R' = \sqrt{R_x{}^2 + R_y{}^2} = \sqrt{(18.0\text{ m})^2 + (25.0\text{ m})^2} = \underline{30.8\text{ m}}$$
$$\theta = \tan^{-1}\frac{\text{opp}}{\text{adj}} = \tan^{-1}\left(\frac{25.0\text{ m}}{18.0\text{ m}}\right) = \underline{54.2°\text{ S of W}}$$



(b) Now do the same calculation, except walk 25.0 m to the north, followed by 18.0 m to the east. Use the equations $R_x = A_x + B_x$ and $R_y = A_y + B_y$:

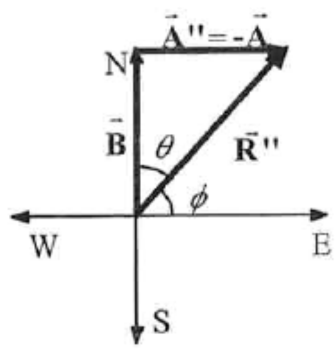
$R_x = 18.0\ \text{m}, R_y = 25.0\ \text{m}$

Then, use the equations $R = \sqrt{R_x^2 + R_y^2}$ and $\theta = \tan^{-1}\left(R_x / R_y\right)$.

$$R'' = \sqrt{R_x^{\ 2} + R_y^{\ 2}} = \sqrt{(18.0\ \text{m})^2 + (25.0\ \text{m})^2} = \underline{30.8\ \text{m}}$$

$$\theta = \tan^{-1}\frac{\text{opp}}{\text{adj}} = \tan^{-1}\left(\frac{25.0\ \text{m}}{18.0\ \text{m}}\right) = \underline{54.2°\ \text{N of E}}$$

which is consistent with part (a).



3.4 PROJECTILE MOTION

30. *A rugby player passes the ball 7.00 m across the field, where it is caught at the same height as it left his hand. (a) At what angle was the ball thrown if its initial speed was 12.0 m/s, assuming that the smaller of the two possible angles was used? (b) What other angle gives the same range, and why would it not be used? (c) How long did this pass take?*

Solution (a) Find the range of a projectile on level ground for which air resistance is negligible:

$R = \dfrac{v_0^{\ 2} \ \sin 2\theta_0}{g}$, where v_0 is the initial speed and θ_0 is the initial angle relative to the horizontal. Solving for initial angle gives:

$\theta_0 = \dfrac{1}{2}\sin^{-1}\left(\dfrac{\text{gR}}{v_0^{\ 2}}\right)$, where: $R = 7.0\ \text{m}, v_0 = 12.0\ \text{m/s}, \text{and}\ g = 9.8\ \text{m/s}^2$.

Therefore, $\theta_0 = \dfrac{1}{2}\sin^{-1}\left(\dfrac{(9.80\ \text{m/s}^2)(7.0\ \text{m})}{(12.0\ \text{m/s})^2}\right) = \underline{14.2°}$

(b) Looking at the equation $R = \dfrac{v_0^{\ 2} \ \sin 2\theta_0}{g}$, we see that range will be same for another angle, θ_0', where $\theta_0 + \theta_0' = 90°$ or $\theta_0' = 90° - 14.2° = \underline{75.8°}$.

This angle is not used as often, because the time of flight will be longer. In rugby that means the defense would have a greater time to get into position to knock down or intercept the pass that has the larger angle of release.

40. *An eagle is flying horizontally at a speed of 3.00 m/s when the fish in her talons wiggles loose and falls into the lake 5.00 m below. Calculate the velocity of the fish relative to the water when it hits the water.*

Solution <u>*x*-direction (horizontal)</u>

Given: $v_{0,x} = 3.00\ \text{m/s}, a_x = 0\ \text{m/s}^2$.

Calculate v_x.

$$v_x = v_{0,x} = \text{constant} = 3.00\text{ m/s}$$

y-direction (vertical)

Given: $v_{0,y} = 0.00\text{ m/s}, a_y = -g = -9.80\text{ m/s}^2, \Delta y = (y - y_0) = -5.00\text{ m}$

Calculate v_y.

$$v_y^2 = v_{0,y'}^2 - 2g(y\text{-}y_0)$$
$$v_y = \sqrt{(0\text{ m/s})^2 - 2(9.80\text{ m/s}^2)(-5.00\text{ m})} = -9.90\text{ m/s}$$

Now we can calculate the final velocity:

$$v = \sqrt{v_x{}^2 + v_y{}^2} = \sqrt{(3.00\text{ m/s})^2 + (-9.90\text{ m/s})^2} = 10.3\text{ m/s}$$

and $\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{-9.90\text{ m/s}}{3.00\text{ m/s}}\right) = -73.1°$

so that $v = \underline{10.3\text{ m/s}, 73.1°\text{ below the horizontal}}$

46. *A basketball player is running at* 5.00 m/s *directly toward the basket when he jumps into the air to dunk the ball. He maintains his horizontal velocity. (a) What vertical velocity does he need to rise 0.750 m above the floor? (b) How far from the basket (measured in the horizontal direction) must he start his jump to reach his maximum height at the same time as he reaches the basket?*

Solution (a) Given: $v_x = 5.00\text{ m/s}, y - y_0 = 0.75\text{ m}, v_y = 0\text{ m/s}, a_y = -g = -9.80\text{ m/s}^2$.

Find: $v_{0,y}$.

Using the equation $v_y{}^2 = v_y{}^2 - 2g(y - y_0)$ gives:

$$v_{0,y} = \sqrt{v_y^{\ 2} + 2g(y - y_0)} = \sqrt{(0\ \text{m/s})^2 + 2(9.80\ \text{m/s}^2)(0.75\ \text{m})} = \underline{3.83\ \text{m/s}}$$

(b) To calculate the x-direction information, remember that the time is the same in the x- and y-directions. Calculate the time from the y-direction information, then use it to calculate the x-direction information:

Calculate the time:

$v_y = v_{0,y} - gt$, so that

$$t = \frac{v_{0,y} - v_y}{g} = \frac{(3.83\ \text{m/s}) - (0\ \text{m/s})}{9.80\ \text{m/s}^2} = 0.391\ \text{s}$$

Now, calculate the horizontal distance he travels to the basket:

$$x = x_0 + v_x t, \text{ so that } (x - x_0) = v_x t = (5.00\ \text{m/s})(0.391\ \text{s}) = \underline{1.96\ \text{m}}$$

So, he must leave the ground 1.96 m before the basket to be at his maximum height when he reaches the basket.

3.5 ADDITION OF VELOCITIES

54. *Near the end of a marathon race, the first two runners are separated by a distance of 45.0 m. The front runner has a velocity of 3.50 m/s, and the second a velocity of 4.20 m/s. (a) What is the velocity of the second runner relative to the first? (b) If the front runner is 250 m from the finish line, who will win the race, assuming they run at constant velocity? (c) What distance ahead will the winner be when she crosses the finish line?*

Solution (a) To keep track of the runners, let's label F for the first runner and S for the second runner. Then we are given: $v_F = 3.50\ \text{m/s}, v_S = 4.20\ \text{m/s}$. To calculate the velocity of the second runner relative to the first, subtract the velocities:

$$v_{SF} = v_S - v_F = 4.20\ \text{m/s} - 3.50\ \text{m/s} = \underline{0.70\ \text{m/s}}$$ faster than first runner

(b) Use the definition of velocity to calculate the time for each runner separately. For the first runner, she runs 250 m at a velocity of 3.50 m/s:

$$t_{\text{F}} = \frac{x_{\text{F}}}{v_{\text{F}}} = \frac{250\ \text{m}}{3.50\ \text{m/s}} = 71.43\ \text{s}$$

For the second runner, she runs 45 m father than the first runner at a velocity of 4.20 m/s: $t_{\text{S}} = \frac{x_{\text{S}}}{v_{\text{S}}} = \frac{250 + 45\ \text{m}}{4.20\ \text{m/s}} = 70.24\ \text{s}$

So, since $t_{\text{S}} < t_{\text{F}}$, the second runner will win.

(c) We can calculate their relative position, using their relative velocity and time of travel. Initially, the second runner is 45 m behind, the relative velocity was found in part (a), and the time is the time for the second runner, so:

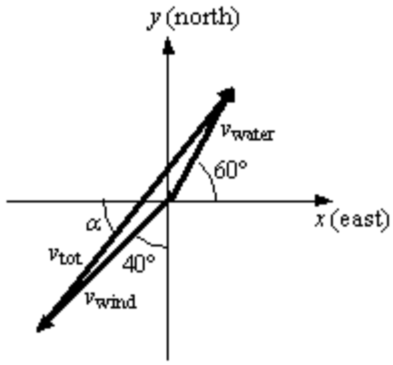
$$x_{\text{SF}} = x_{\text{O,SF}} + v_{\text{SF}} t_{\text{S}} = -45.0\ \text{m} + (0.70\ \text{m/s})(70.24\ \text{s}) = \underline{4.17\ \text{m}}$$

62. *The velocity of the wind relative to the water is crucial to sailboats. Suppose a sailboat is in an ocean current that has a velocity of 2.20 m/s in a direction* 30.0° *east of north relative to the Earth. It encounters a wind that has a velocity of 4.50 m/s in a direction of* 50.0° *south of west relative to the Earth. What is the velocity of the wind relative to the water?*

Solution In order to calculate the velocity of the wind relative to the ocean, we need to add the vectors for the wind and the ocean together, being careful to use vector addition. The velocity of the wind relative to the ocean is equal to the velocity of the wind relative to the earth plus the velocity of the earth relative to the ocean. Now,

$$\mathbf{v}_{\mathbf{WO}} = \mathbf{v}_{\mathbf{WE}} + \mathbf{v}_{\mathbf{EO}} = \mathbf{v}_{\mathbf{WE}} - \mathbf{v}_{\mathbf{OE}}.$$

The first subscript is the object, the second is what it is relative to. In other words the velocity of the earth relative to the ocean is the opposite of the velocity of the ocean relative to the earth.



To solve this vector equation, we need to add the *x*- and *y*-components separately.

$$v_{WOx} = v_{WEx} - v_{OEx} = (-4.50\text{ m/s})\cos 50° - (2.20\text{ m/s})\cos 60° = -3.993\text{ m/s}$$
$$v_{WOy} = v_{WEy} - v_{OEy} = (-4.50\text{ m/s})\sin 50° - (2.20\text{ m/s})\sin 60° = -5.352\text{ m/s}$$

Finally, we can use the equations below to calculate the velocity of the water relative to the ocean:

$$v = \sqrt{v_x^{\ 2} + v_y^{\ 2}} = \sqrt{(-3.993\text{ m/s})^2 + (-5.352\text{ m/s})^2} = \underline{6.68\text{ m/s}}$$
$$\alpha = \tan^{-1}\frac{|v_y|}{|v_x|} = \tan^{-1}\left(\frac{5.352\text{ m/s}}{3.993\text{ m/s}}\right) = \underline{53.3°\text{ S of W}}$$

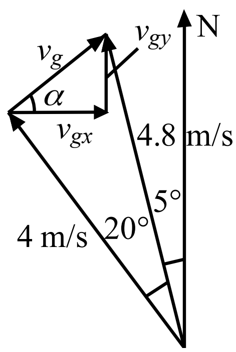
66. *A ship sailing in the Gulf Stream is heading* $25.0°$ *west of north at a speed of 4.00 m/s relative to the water. Its velocity relative to the Earth is* 4.80 m/s $5.00°$ *west of north. What is the velocity of the Gulf Stream? (The velocity obtained is typical for the Gulf Stream a few hundred kilometers off the east coast of the United States.)*

Solution To calculate the velocity of the water relative to the earth, we need to add the vectors. The velocity of the water relative to the earth is equal to the velocity of the water relative to the ship plus the velocity of the ship relative to the earth.

$$\mathbf{v}_{\mathbf{WE}} = \mathbf{v}_{\mathbf{WS}} + \mathbf{v}_{\mathbf{SE}} = -\mathbf{v}_{\mathbf{SW}} + \mathbf{v}_{\mathbf{SE}}$$

Now, we need to calculate the *x*- and *y*-components separately:

$$v_{\text{WE}x} = -v_{\text{SW}x} + v_{\text{SE}x} = -(4.00\text{ m/s})\cos 115° + (4.80\text{ m/s})\cos 95° = 1.272\text{ m/s}$$
$$v_{\text{WE}y} = -v_{\text{SW}y} + v_{\text{SE}y} = -(4.00\text{ m/s})\sin 115° + (4.80\text{ m/s})\sin 95° = 1.157\text{ m/s}$$



Finally, we use the equations below to calculate the velocity of the water relative to the earth:

$$v_{\text{WE}} = \sqrt{v^2{}_{\text{WE},x} + v^2{}_{\text{WE},y}} = \sqrt{(1.272\text{ m/s})^2 + (1.157\text{ m/s})^2} = \underline{1.72\text{ m/s}}$$

$$\alpha = \tan^{-1}\left(\frac{v_{\text{WE},y}}{v_{\text{WE},x}}\right) = \tan^{-1}\frac{1.157\text{ m/s}}{1.272\text{ m/s}} = \underline{42.3°\text{ N of E.}}$$

CHAPTER 4: DYNAMICS: FORCE AND NEWTON'S LAWS OF MOTION

4.3 NEWTON'S SECOND LAW OF MOTION: CONCEPT OF A SYSTEM

1. *A 63.0-kg sprinter starts a race with an acceleration of* $4.20\ \text{m/s}^2$*. What is the net external force on him?*

Solution The net force acting on the sprinter is given by
$\text{net } F = ma = (63.0\ \text{kg})(4.20\ \text{m/s}^2) = \underline{265\ \text{N}}$

7. *(a) If the rocket sled shown in Figure 4.31 starts with only one rocket burning, what is its acceleration? Assume that the mass of the system is 2100 kg, and the force of friction opposing the motion is known to be 650 N. (b) Why is the acceleration not one-fourth of what it is with all rockets burning?*

Solution (a) Use the thrust given for the rocket sled in Figure 4.8, $T = 2.59\times10^4\ \text{N}$. With only one rocket burning, $\text{net } F = T - f$ so that Newton's second law gives:

$$a = \frac{\text{net } F}{m} = \frac{T-f}{m} = \frac{2.59\times10^4\ \text{N} - 650\ \text{N}}{2100\ \text{kg}} = \underline{12.0\ \text{m/s}^2}$$

(b) The acceleration is not one-fourth of what it was with all rockets burning because the frictional force is still as large as it was with all rockets burning.

13. *The weight of an astronaut plus his space suit on the Moon is only 250 N. How much do they weigh on Earth? What is the mass on the Moon? On Earth?*

Solution $w_{\text{Moon}} = mg_{\text{Moon}}$

$$m = \frac{w_{\text{Moon}}}{g_{\text{Moon}}} = \frac{250\text{ N}}{1.67\text{ m/s}^2} = 150\text{ kg}$$

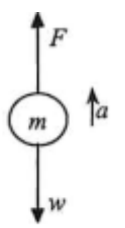
$$w_{\text{Earth}} = mg_{\text{Earth}} = (150\text{ kg})(9.8\text{ m/s}^2) = 1470\text{ N} = \underline{1.5\times10^3\text{ N}}$$

Mass does not change. The astronaut's mass on both Earth and the Moon is 150 kg.

4.6 PROBLEM-SOLVING STRATEGIES

25. *Calculate the force a 70.0-kg high jumper must exert on the ground to produce an upward acceleration 4.00 times the acceleration due to gravity. Explicitly show how you follow the steps in the Problem-Solving Strategy for Newton's laws of motion.*

Solution Step 1. Use Newton's Laws of Motion.



Force acting on the high jumper.

Step 2. Given: $a = 4.00\,g = (4.00)(9.80\text{ m/s}^2) = 39.2\text{ m/s}^2$; $m = 70.0\text{ kg}$

Find F .

Step 3. $\sum F = +F - w = ma$, so that $F = ma + w = ma + mg = m(a+g)$

$$F = (70.0\text{ kg})[(39.2\text{ m/s}^2) + (9.80\text{ m/s}^2)] = \underline{3.43\times10^3\text{ N}}$$

The force exerted by the high-jumper is actually down on the ground, but $\mathbf{F}$ is up from the ground to help him jump.

Step 4. This result is reasonable, since it is quite possible for a person to exert a force of the magnitude of $10^3\ \text{N}$.

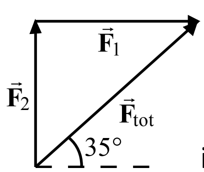
30. *(a) Find the magnitudes of the forces $\mathbf{F}_1$ and $\mathbf{F}_2$ that add to give the total force $\mathbf{F}_{\text{tot}}$ shown in Figure 4.35. This may be done either graphically or by using trigonometry. (b) Show graphically that the same total force is obtained independent of the order of addition of $\mathbf{F}_1$ and $\mathbf{F}_2$. (c) Find the direction and magnitude of some other pair of vectors that add to give $\mathbf{F}_{\text{tot}}$. Draw these to scale on the same drawing used in part (b) or a similar picture.*

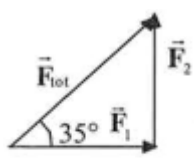
Solution (a) Since F_2 is the y-component of the total force:

$$F_2 = F_{\text{tot}} \sin 35° = (20\ \text{N})\sin 35° = 11.47\ \text{N} = \underline{11\ \text{N}}.$$

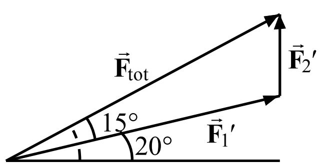
And F_1 is the x-component of the total force:

$$F_1 = F_{\text{tot}} \cos 35° = (20\ \text{N})\cos 35° = 16.38\ \text{N} = \underline{16\ \text{N}}.$$

(b) 

is the same as: 

(c) For example, use vectors as shown in the figure.



F_1' is at an angle of $20°$ from the horizontal, with a magnitude of $F_1' \cos 20° = F_1$

$$F_1' = \frac{F_1}{\cos 20°} = \frac{16.38\ \text{N}}{\cos 20°} = 17.4\ \text{N} = \underline{17\ \text{N}}$$

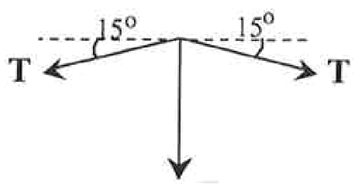
F_2' is at an angle of $90°$ from the horizontal, with a magnitude of

$$F_2' = F_2 - F_1' \sin 20° = \underline{5.2\ \text{N}}$$

33. *What force is exerted on the tooth in Figure 4.38 if the tension in the wire is 25.0 N? Note that the force applied to the tooth is smaller than the tension in the wire, but this is necessitated by practical considerations of how force can be applied in the mouth. Explicitly show how you follow steps in the Problem-Solving Strategy for Newton's laws of motion.*

Solution Step 1: Use Newton's laws since we are looking for forces.

Step 2: Draw a free body diagram:



Step 3: Given $T = 25.0\ \text{N}$, find F_{app}. Using Newton's laws gives $\Sigma F_y = 0$, so that the applied force is due to the y-components of the two tensions:

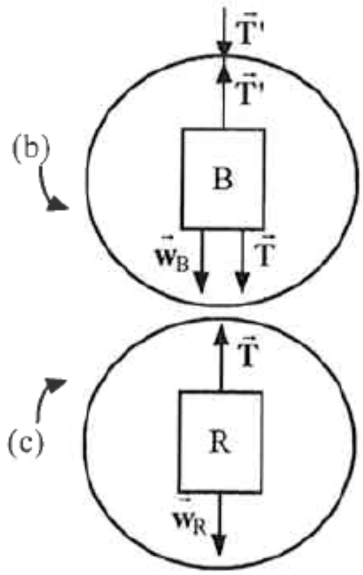
$F_{\text{app}} = 2T\sin\theta = 2(25.0\ \text{N})\sin 15° = \underline{12.9\ \text{N}}$

The x-components of the tension cancel. $\sum F_x = 0$

Step 4: This seems reasonable, since the applied tensions should be greater than the force applied to the tooth.

34. *Figure 4.39 shows Superhero and Trusty Sidekick hanging motionless from a rope. Superhero's mass is 90.0 kg, while Trusty Sidekick's is 55.0 kg, and the mass of the rope is negligible. (a) Draw a free-body diagram of the situation showing all forces acting on Superhero, Trusty Sidekick, and the rope. (b) Find the tension in the rope above Superhero. (c) Find the tension in the rope between Superhero and Trusty Sidekick. Indicate on your free-body diagram the system of interest used to solve each part.*

Solution (a)



(b) Using the upper circle of the diagram, $\sum F_y = 0$, so that $T' - T - w_B = 0$.

Using the lower circle of the diagram, $\sum F_y = 0$, giving $T - w_R = 0$.

Next, write the weights in terms of masses: $w_B = m_B g$, $w_R = m_R g$.

Solving for the tension in the upper rope gives:

$T' = T + w_B = w_R + w_B = m_R g + m_B g = g(m_R + m_B)$

Plugging in the numbers gives: $T' = (9.80 \text{ m/s}^2)(55.0 \text{ kg} + 90.0 \text{ kg}) = \underline{1.42 \times 10^3 \text{ N}}$

Using the lower circle of the diagram, net $\sum F_y = 0$, so that $T - w_R = 0$. Again, write the weight in terms of mass: $w_R = m_R g$. Solving for the tension in the lower rope gives: $T = m_R g = (55.0 \text{ kg})(9.80 \text{ m/s}^2) = \underline{539 \text{ N}}$

4.7 FURTHER APPLICATIONS OF NEWTON'S LAWS OF MOTION

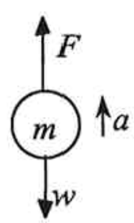
46. ***Integrated Concepts*** *A basketball player jumps straight up for a ball. To do this, he lowers his body 0.300 m and then accelerates through this distance by forcefully straightening his legs. This player leaves the floor with a vertical velocity sufficient to carry him 0.900 m above the floor. (a) Calculate his velocity when he leaves the floor. (b) Calculate his acceleration while he is straightening his legs. He goes from zero to the velocity found in part (a) in a distance of 0.300 m. (c) Calculate the force he exerts on the floor to do this, given that his mass is 110 kg.*

Solution (a) After he leaves the ground, the basketball player is like a projectile. Since he reaches a maximum height of 0.900 m, $v^2 = v_0^2 - 2g(y - y_0)$, with $y - y_0 = 0.900\text{ m}, \text{and } v = 0\text{ m/s}.$ Solving for the initial velocity gives:

$$v_0 = [2g(y - y_0)]^{1/2} = [2(9.80\text{ m/s}^2)(0.900\text{ m})]^{1/2} = \underline{4.20\text{ m/s}}$$

(b) Since we want to calculate his acceleration, use $v^2 = v_0^2 + 2a(y - y_0)$, where $y - y_0 = 0.300\text{ m},$ and since he starts from rest, $v_0 = 0\text{ m/s}.$ Solving for the acceleration gives: $a = \dfrac{v^2}{2(y - y_0)} = \dfrac{(4.20\text{ m/s})^2}{(2)(0.300\text{ m})} = \underline{29.4\text{ m/s}^2}$

(c)



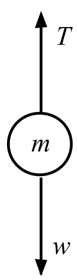
Now, we must draw a free body diagram in order to calculate the force exerted by the basketball player to jump. The net force is equal to the mass times the acceleration: $\text{net } F = ma = F - w = F - mg$

So, solving for the force gives:

$$F = ma + mg = m(a + g) = 110\text{ kg}(29.4\text{ m/s}^2 + 9.80\text{ m/s}^2) = \underline{4.31\times10^3\text{ N}}$$

49. ***Integrated Concepts*** *An elevator filled with passengers has a mass of 1700 kg. (a) The elevator accelerates upward from rest at a rate of* $1.20\ \text{m/s}^2$ *for 1.50 s. Calculate the tension in the cable supporting the elevator. (b) The elevator continues upward at constant velocity for 8.50 s. What is the tension in the cable during this time? (c) The elevator decelerates at a rate of* $0.600\ \text{m/s}^2$ *for 3.00 s. What is the tension in the cable during deceleration? (d) How high has the elevator moved above its original starting point, and what is its final velocity?*

Solution (a)



The net force is due to the tension and the weight:

$\text{net } F = ma = T - w = T - mg, \text{ and } m = 1700\ \text{kg}.$

$a = 1.20\ \text{m/s}^2, \text{ so the tension is:}$

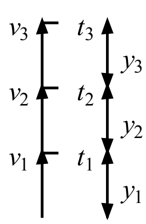
$T = m(a+g) = (1700\ \text{kg})(1.20\ \text{m/s}^2 + 9.80\ \text{m/s}^2) = \underline{1.87\times10^4\ \text{N}}$

(b) $a = 0\ \text{m/s}^2$, so the tension is: $T = w = mg = (1700\ \text{kg})(9.80\ \text{m/s}^2) = \underline{1.67\times10^4\ \text{N}}$

(c) $a = 0.600\ \text{m/s}^2, \text{ but down:}$

$T = m(g-a) = (1700\ \text{kg})(9.80\ \text{m/s}^2 - 0.600\ \text{m/s}^2) = \underline{1.56\times10^4\ \text{N}}$

(d)



Use $y - y_0 = v_0 t + \frac{1}{2}at^2$ and $v = v_0 + at.$

For part (a), $v_0 = 0\text{ m/s},\ a = 1.20\text{ m/s}^2, t = 150\text{ s},$ given

$$y_1 = \frac{1}{2}a_1t_1^2 = \frac{1}{2}(1.20\text{ m/s}^2)(1.50\text{ s})^2 = 1.35\text{ m}$$ and

$$v_1 = a_1t_1 = (1.20\text{ m/s}^2)(1.50\text{ s}) = 1.80\text{ m/s}.$$

For part (b), $v_0 = v = 1.80\text{ m/s}, a = 0\text{ m/s}, t = 8.50\text{ s},$ so

$$y_1 = v_1t_2 = (1.80\text{ m/s})(8.50\text{ s}) = 15.3\text{ m}.$$

For part (c), $v_0 = 1.80\text{ m/s}, a = -0.600\text{ m/s}^2, t = 3.00\text{ s}$, so that:

$$y_3 = v_2t + a_3t_3^2 = (1.80\text{ m/s})(3.00\text{ s}) + 0.5(-0.600\text{ m/s})(3.00\text{ s})^2 = 2.70\text{ m}$$

$$v_3 = v_2 + a_3t_3 = 1.80\text{ m/s} + (-0.600\text{ m/s}^2)(3.00\text{ s}) = 0\text{ m/s}$$

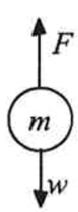
Finally, the total distance traveled is

$$y_1 + y_2 + y_3 = 1.35\text{ m} + 15.3\text{ m} + 2.70\text{ m} = 19.35\text{ m} = \underline{19.4\text{ m}}$$

And the final velocity will be the velocity at the end of part (c), or $v_{\text{final}} = \underline{0\text{ m/s}}$.

51. ***Unreasonable Results*** *A 75.0-kg man stands on a bathroom scale in an elevator that accelerates from rest to 30.0 m/s in 2.00 s. (a) Calculate the scale reading in newtons and compare it with his weight. (The scale exerts an upward force on him equal to its reading.) (b) What is unreasonable about the result? (c) Which premise is unreasonable, or which premises are inconsistent?*

Solution (a)



Using $v = v_o + at$ gives: $a = \dfrac{v - v_0}{t} = \dfrac{30.0\text{ m/s} - 0\text{ m/s}}{2.00\text{ s}} = 15.0\text{ m/s}^2$.

Now, using Newton's laws gives $\text{net } F = F - w = ma$, so that

$$F = m(a+g) = 75.0\ \text{kg}\left(15.0\ \text{m/s}^2 + 9.80\ \text{m/s}^2\right) = \underline{1860\ \text{N}}.$$

The ratio of the force to the weight is then:

$$\frac{F}{w} = \frac{m(a+g)}{mg} = \frac{15.0\ \text{m/s}^2 + 9.80\ \text{m/s}^2}{9.80\ \text{m/s}^2} = \underline{2.53}$$

(b) The value (1860 N) is more force than you expect to experience on an elevator.

(c) The acceleration $a = 15.0\ \text{m/s}^2 = 1.53g$ is much higher than any standard elevator. The final speed is too large (30.0 m/s is VERY fast)! The time of 2.00s is not unreasonable for an elevator.

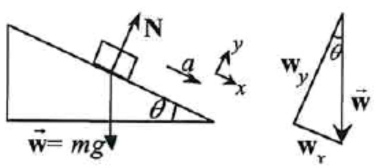
CHAPTER 5: FURTHER APPLICATION OF NEWTON'S LAWS: FRICTION, DRAG, AND ELASTICITY

5.1 FRICTION

8. *Show that the acceleration of any object down a frictionless incline that makes an angle* θ *with the horizontal is* $a = g\sin\theta$*. (Note that this acceleration is independent of mass.)*

Solution The component of w down the incline leads to the acceleration:

$w_x = \text{net } F_x = ma = mg\sin\theta$ so that $\underline{a = g\sin\theta}$

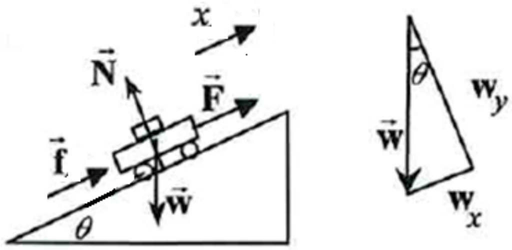


The component of w perpendicular to the incline equals the normal force.

$w_y = \text{net } F_y = 0 = N - mg\sin\theta$

14. *Calculate the maximum acceleration of a car that is heading up a* $4°$ *slope (one that makes an angle of* $4°$ *with the horizontal) under the following road conditions. Assume that only half the weight of the car is supported by the two drive wheels and that the coefficient of static friction is involved—that is, the tires are not allowed to slip during the acceleration. (Ignore rolling.) (a) On dry concrete. (b) On wet concrete. (c) On ice, assuming that* $\mu_s = 0.100$*, the same as for shoes on ice.*

Solution



Take the positive *x*-direction as up the slope. For max acceleration,

$$\text{net } F_x = ma = f - w_x = \frac{1}{2}\mu_s mg\cos\theta - mg\sin\theta$$

So the maximum acceleration is: $a = g\left(\frac{1}{2}\mu_s\cos\theta - \sin\theta\right)$

(a) $\mu_s = 1.00,\ a = (9.80\ \text{m/s}^2)\left[\frac{1}{2}(1.00)\cos4° - \sin4°\right] = \underline{4.20\ \text{m/s}^2}$

(b) $\mu_s = 0.700,\ a = (9.80\ \text{m/s}^2)\left[\frac{1}{2}(0.700)\cos4° - \sin4°\right] = \underline{2.74\ \text{m/s}^2}$

(c) $\mu_s = 0.100,\ a = (9.80\ \text{m/s}^2)\left[\frac{1}{2}(0.100)\cos4° - \sin4°\right] = \underline{-0.195\ \text{m/s}^2}$

The negative sign indicates downwards acceleration, so the car cannot make it up the grade.

5.3 ELASTICITY: STRESS AND STRAIN

29. *During a circus act, one performer swings upside down hanging from a trapeze holding another, also upside-down, performer by the legs. If the upward force on the lower performer is three times her weight, how much do the bones (the femurs) in her upper legs stretch? You may assume each is equivalent to a uniform rod 35.0 cm long and 1.80 cm in radius. Her mass is 60.0 kg.*

Solution Use the equation $\Delta L = \frac{1}{Y}\frac{F}{A}L_0$, where $Y = 1.6\times10^{10}\ \text{N/m}^2$ (from Table 5.3), $L_0 = 0.350\ \text{m},\ A = \pi r^2 = \pi(0.0180\ \text{m})^2 = 1.018\times10^{-3}\ \text{m}^2$, and

$F_{\text{tot}} = 3w = 3(60.0\text{ kg})(9.80\text{ m/s}^2) = 1764\text{ N}$, so that the force on each leg is $F_{\text{leg}} = F_{\text{tot}}/2 = 882\text{ N}$. Substituting in the value gives:

$$\Delta L = \frac{1}{1.6\times10^{10}\text{ N/m}^2}\frac{(882\text{ N})}{(1.018\times10^{-3}\text{ m}^2)}(0.350\text{ m}) = 1.90\times10^{-5}\text{ m}.$$

So each leg is stretched by $\underline{1.90\times10^{-3}\text{ cm}}$.

35. *As an oil well is drilled, each new section of drill pipe supports its own weight and that of the pipe and drill bit beneath it. Calculate the stretch in a new 6.00 m length of steel pipe that supports 3.00 km of pipe having a mass of 20.0 kg/m and a 100-kg drill bit. The pipe is equivalent in strength to a solid cylinder 5.00 cm in diameter.*

Solution Use the equation $\Delta L = \frac{1}{Y}\frac{F}{A}L_0$, where $L_0 = 6.00\text{ m}, Y = 1.6\times10^{10}\text{ N/m}^2$. To calculate the mass supported by the pipe, we need to add the mass of the new pipe to the mass of the 3.00 km piece of pipe and the mass of the drill bit:

$$m = m_{\text{p}} + m_{3\text{ km}} + m_{\text{bit}}$$
$$= (6.00\text{ m})(20.0\text{ kg/m}) + (3.00\times10^3\text{ m})(20.0\text{ kg/m}) + 100\text{ kg} = 6.022\times10^4\text{ kg}$$

So that the force on the pipe is:
$$F = w = mg = (6.022\times10^4\text{ kg})(9.80\text{ m/s}^2) = 5.902\times10^5\text{ N}$$

Finally the cross sectional area is given by: $A = \pi r^2 = \pi\left(\frac{0.0500\text{ m}}{2}\right)^2 = 1.963\times10^{-3}\text{ m}^2$

Substituting in the values gives:
$$\Delta L = \frac{1}{2.10\times10^{11}\text{ N/m}^2}\frac{(5.902\times10^5\text{ N})}{(1.963\times10^{-3}\text{ m}^2)}(6.00\text{ m}) = 8.59\times10^{-3}\text{ m} = \underline{8.59\text{ mm}}$$

41. *A farmer making grape juice fills a glass bottle to the brim and caps it tightly. The juice expands more than the glass when it warms up, in such a way that the volume increases by 0.2% (that is, $\Delta V/V_0 = 2\times10^{-3}$) relative to the space available. Calculate the force exerted by the juice per square centimeter if its bulk modulus is $1.8\times10^9\text{ N/m}^2$, assuming the bottle does not break. In view of your answer, do you think the bottle will survive?*

Solution Using the equation $\Delta V = \frac{1}{B}\frac{F}{A}V_0$ gives:

$$\frac{F}{A} = B\frac{\Delta V}{V_0} = \left(1.8\times10^9\ \text{N/m}^2\right)\left(2\times10^{-3}\right) = 3.6\times10^6\ \text{N/m}^2 = \underline{4\times10^6\ \text{N/m}^2} = 4\times10^2\ \text{N/cm}^2$$

Since $1\ \text{atm} = 1.013\times10^5\ \text{N/m}^2$, the pressure is about 36 atmospheres, far greater than the average jar is designed to withstand.

CHAPTER 6: UNIFORM CIRCULAR MOTION AND GRAVITATION

6.1 ROTATION ANGLE AND ANGULAR VELOCITY

1. *Semi-trailer trucks have an odometer on one hub of a trailer wheel. The hub is weighted so that it does not rotate, but it contains gears to count the number of wheel revolutions—it then calculates the distance traveled. If the wheel has a 1.15 m diameter and goes through 200,000 rotations, how many kilometers should the odometer read?*

Solution Given:

$$d = 1.15\text{ m} \Rightarrow r = \frac{1.15\text{ m}}{2} = 0.575\text{ m}, \Delta\theta = 200{,}000\text{ rot} \times \frac{2\pi\text{ rad}}{1\text{ rot}} = 1.257\times10^{6}\text{ rad}$$

Find Δs using $\Delta\theta = \frac{\Delta s}{r}$, so that

$$\Delta s = \Delta\theta \times r = (1.257\times10^{6}\text{ rad})(0.575\text{ m}) = 7.226\times10^{5}\text{ m} = \underline{723\text{ km}}$$

7. *A truck with 0.420 m radius tires travels at 32.0 m/s. What is the angular velocity of the rotating tires in radians per second? What is this in rev/min?*

Solution Given: $r = 0.420\text{ m}, v = 32.0\text{ m/s}$.

Use $\omega = \frac{v}{r} = \frac{32.0\text{ m/s}}{0.420\text{ m}} = \underline{76.2\text{ rad/s}}.$

Convert to rpm by using the conversion factor:

$1\text{ rev} = 2\pi\text{ rad}$,

$$\omega = 76.2\text{ rad/s} \times \frac{1\text{ rev}}{2\pi\text{ rad}} \times \frac{60\text{ s}}{1\text{ min}}$$
$$= 728\text{ rev/s} = \underline{728\text{ rpm}}$$

6.2 CENTRIPETAL ACCELERATION

18. *Verify that the linear speed of an ultracentrifuge is about 0.50 km/s, and Earth in its orbit is about 30 km/s by calculating: (a) The linear speed of a point on an ultracentrifuge 0.100 m from its center, rotating at 50,000 rev/min. (b) The linear speed of Earth in its orbit about the Sun (use data from the text on the radius of Earth's orbit and approximate it as being circular).*

Solution (a) Use $v = r\omega$ to find the linear velocity:

$$v = r\omega = (0.100\text{ m})\left(50{,}000\ \text{ rev/min} \times \frac{2\,\pi\text{ rad}}{1\text{ rev}} \times \frac{1\text{ min}}{60\text{ s}}\right) = 524\text{ m/s} = \underline{0.524\text{ km/s}}$$

(b) Given: $\omega = 2\pi\frac{\text{rad}}{\text{y}} \times \frac{1\text{ y}}{3.16\times 10^{7}\text{ s}} = 1.988\times 10^{-7}\text{ rad/s}; r = 1.496\times 10^{11}\text{ m}$

Use $v = r\omega$ to find the linear velocity:

$$v = r\omega = (1.496\times 10^{11}\text{ m})(1.988\times 10^{-7}\text{ rad/s}) = 2.975\times 10^{4}\text{ m/s} = \underline{29.7\text{ km/s}}$$

6.3 CENTRIPETAL FORCE

26. *What is the ideal speed to take a 100 m radius curve banked at a 20.0° angle?*

Solution Using $\tan\theta = \frac{v^2}{rg}$ gives:

$$\tan\theta = \frac{v^2}{rg} \Rightarrow v = \sqrt{rg\tan\theta} = \sqrt{(100\text{ m})(9.8\text{ m/s}^2)\tan 20.0°} = \underline{18.9\text{ m/s}}$$

6.5 NEWTON'S UNIVERSAL LAW OF GRAVITATION

33. *(a) Calculate Earth's mass given the acceleration due to gravity at the North Pole is* 9.830 m/s^2 *and the radius of the Earth is 6371 km from pole to pole. (b) Compare this with the accepted value of* $5.979\times10^{24}\text{ kg}$.

Solution (a) Using the equation $g = \frac{GM}{r^2}$ gives:

$$g = \frac{GM}{r^2} \Rightarrow M = \frac{r^2 g}{G} = \frac{(6371\times10^3\text{ m})^2(9.830\text{ m/s}^2)}{6.673\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2} = \underline{5.979\times10^{24}\text{ kg}}$$

(b) This is identical to the best value to three significant figures.

39. *Astrology, that unlikely and vague pseudoscience, makes much of the position of the planets at the moment of one's birth. The only known force a planet exerts on Earth is gravitational. (a) Calculate the gravitational force exerted on a 4.20 kg baby by a 100 kg father 0.200 m away at birth (he is assisting, so he is close to the child). (b) Calculate the force on the baby due to Jupiter if it is at its closest distance to Earth, some* $6.29\times10^{11}\text{ m}$ *away. How does the force of Jupiter on the baby compare to the force of the father on the baby? Other objects in the room and the hospital building also exert similar gravitational forces. (Of course, there could be an unknown force acting, but scientists first need to be convinced that there is even an effect, much less that an unknown force causes it.)*

Solution (a) Use $F = \dfrac{GMm}{r^2}$ to calculate the force:

$$F_\text{f} = \frac{GMm}{r^2} = \frac{(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(100\ \text{kg})(4.20\ \text{kg})}{(0.200\ \text{m})^2} = \underline{7.01\times10^{-7}\ \text{N}}$$

(b) The mass of Jupiter is:

$$m_\text{J} = 1.90\times10^{27}\ \text{kg}$$

$$F_\text{J} = \frac{(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(1.90\times10^{27}\ \text{kg})(4.20\ \text{kg})}{(6.29\times10^{11}\ \text{m})^2} = \underline{1.35\times10^{-6}\ \text{N}}$$

$$\frac{F_\text{f}}{F_\text{J}} = \frac{7.01\times10^{-7}\ \text{N}}{1.35\times10^{-6}\ \text{N}} = \underline{0.521}$$

6.6 SATELLITES AND KEPLER'S LAWS: AN ARGUMENT FOR SIMPLICITY

45. *Find the mass of Jupiter based on data for the orbit of one of its moons, and compare your result with its actual mass.*

Solution Using $\dfrac{r^3}{T^2} = \dfrac{G}{4\pi^2}M$, we can solve the mass of Jupiter:

$$M_\text{J} = \frac{4\pi^2}{G}\times\frac{r^3}{T^2}$$

$$= \frac{4\pi^2}{6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2}\times\frac{(4.22\times10^8\ \text{m})^3}{[(0.00485\ \text{y})(3.16\times10^7\ \text{s/y})]^2} = \underline{1.89\times10^{27}\ \text{kg}}$$

This result matches the value for Jupiter's mass given by NASA.

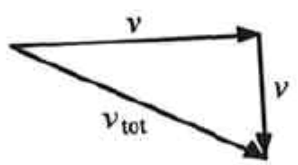
48. ***Integrated Concepts*** *Space debris left from old satellites and their launchers is becoming a hazard to other satellites. (a) Calculate the speed of a satellite in an orbit 900 km above Earth's surface. (b) Suppose a loose rivet is in an orbit of the same*

radius that intersects the satellite's orbit at an angle of 90° *relative to Earth. What is the velocity of the rivet relative to the satellite just before striking it? (c) Given the rivet is 3.00 mm in size, how long will its collision with the satellite last? (d) If its mass is 0.500 g, what is the average force it exerts on the satellite? (e) How much energy in joules is generated by the collision? (The satellite's velocity does not change appreciably, because its mass is much greater than the rivet's.)*

Solution (a) Use $F_c = ma_c$, then substitute using $a = \frac{v^2}{r}$ and $F = \frac{GmM}{r^2}$.

$$\frac{GmM}{r^2} = \frac{mv^2}{r}$$

$$v = \sqrt{\frac{GM_E}{r_S}} = \sqrt{\frac{(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.979\times10^{24}\ \text{kg})}{900\times10^3\ \text{m}}} = \underline{2.11\times10^4\ \text{m/s}}$$

(b) 

In the satellite's frame of reference, the rivet has two perpendicular velocity components equal to v from part (a):

$$v_{tot} = \sqrt{v^2 + v^2} = \sqrt{2v^2} = \sqrt{2}(2.105\times10^4\ \text{m/s}) = \underline{2.98\times10^4\ \text{m/s}}$$

(c) Using kinematics: $d = v_{tot}t \Rightarrow t = \frac{d}{v_{tot}} = \frac{3.00\times10^{-3}\ \text{m}}{2.98\times10^4\ \text{m/s}} = \underline{1.01\times10^{-7}\ \text{s}}$

(d) $\overline{F} = \frac{\Delta p}{\Delta t} = \frac{mv_{tot}}{t} = \frac{(0.500\times10^{-3}\ \text{kg})(2.98\times10^4\ \text{m/s})}{1.01\times10^{-7}\ \text{s}} = \underline{1.48\times10^8\ \text{N}}$

(e) The energy is generated from the rivet. In the satellite's frame of reference, $v_i = v_{tot}$, and $v_f = 0$. So, the change in the kinetic energy of the rivet is:

$$\Delta\,\text{KE} = \frac{1}{2}mv_{tot}{}^2 - \frac{1}{2}mv_i{}^2 = \frac{1}{2}(0.500\times10^{-3}\ \text{kg})(2.98\times10^4\ \text{m/s})^2 - 0\ \text{J} = \underline{2.22\times10^5\ \text{J}}$$

CHAPTER 7: WORK, ENERGY, AND ENERGY RESOURCES

7.1 WORK: THE SCIENTIFIC DEFINITION

1. *How much work does a supermarket checkout attendant do on a can of soup he pushes 0.600 m horizontally with a force of 5.00 N? Express your answer in joules and kilocalories.*

Solution Using $W = fd\cos(\theta)$, where $F = 5.00\text{ N}, d = 0.600\text{ m}$ and since the force is applied horizontally, $\theta = 0°$: $W = Fd\cos\theta = (5.00\text{ N})(0.600\text{ m})\cos 0° = \underline{3.00\text{ J}}$

Using the conversion factor $1\text{ kcal} = 4186\text{ J}$ gives:

$$W = 3.00\text{ J} \times \frac{1\text{ kcal}}{4186\text{ J}} = \underline{7.17 \times 10^{-4}\text{ kcal}}$$

7. *A shopper pushes a grocery cart 20.0 m at constant speed on level ground, against a 35.0 N frictional force. He pushes in a direction $25.0°$ below the horizontal. (a) What is the work done on the cart by friction? (b) What is the work done on the cart by the gravitational force? (c) What is the work done on the cart by the shopper? (d) Find the force the shopper exerts, using energy considerations. (e) What is the total work done on the cart?*

Solution (a) The work done by friction is in the opposite direction of the motion, so $\theta = 180°$, and therefore $W_f = Fd\cos\theta = 35.0\text{ N} \times 20.0\text{ m} \times \cos 180° = \underline{-700\text{ J}}$

(b) The work done by gravity is perpendicular to the direction of motion, so $\theta = 90°$, and $W_g = Fd\cos\theta = 35.0\text{ N} \times 20.0\text{ m} \times \cos 90° = \underline{0\text{ J}}$

(c) If the cart moves at a constant speed, no energy is transferred to it, from the work-energy theorem: $\text{net } W = W_s + W_f = 0, \text{ or } W_s = \underline{700\text{ J}}$

(d) Use the equation $W_s = Fd\cos\theta$, where $\theta = 25°$, and solve for the force:

$$F = \frac{W_s}{d\cos\theta} = \frac{700\text{ J}}{20.0\text{ m}\times\cos 25°} = 38.62\text{ N} = \underline{38.6\text{ N}}$$

(e) Since there is no change in speed, the work energy theorem says that there is no net work done on the cart: $\text{net}W = W_f + W_s = -700\text{ J} + 700\text{ J} = \underline{0\text{ J}}$

7.2 KINETIC ENERGY AND THE WORK-ENERGY THEOREM

13. *A car's bumper is designed to withstand a 4.0-km/h (1.1-m/s) collision with an immovable object without damage to the body of the car. The bumper cushions the shock by absorbing the force over a distance. Calculate the magnitude of the average force on a bumper that collapses 0.200 m while bringing a 900-kg car to rest from an initial speed of 1.1 m/s.*

Solution Use the work energy theorem, $\text{net}W = \frac{1}{2}mv^2 - \frac{1}{2}m{v_0}^2 = Fd\cos\theta$,

$$F = \frac{mv^2 - m{v_0}^2}{2d\cos\theta} = \frac{(900\text{ kg})(0\text{ m/s})^2 - (900\text{ kg})(1.12\text{ m/s})^2}{2(0.200\text{ m})\cos 0°} = \underline{-2.8\times10^3\text{ N}}$$

The force is negative because the car is decelerating.

7.3 GRAVITATIONAL POTENTIAL ENERGY

16. *A hydroelectric power facility (see Figure 7.38) converts the gravitational potential energy of water behind a dam to electric energy. (a) What is the gravitational potential energy relative to the generators of a lake of volume* 50.0 km^3 *(* $\text{mass} = 5.00\times10^{13}\text{ kg}$*), given that the lake has an average height of 40.0 m above the generators? (b) Compare this with the energy stored in a 9-megaton fusion bomb.*

Solution (a) Using the equation $\Delta PE_g = mgh$, where

$m = 5.00\times10^{13}\text{ kg}$, $g = 9.80\text{ m/s}^2$, and $h = 40.0\text{ m}$, gives:

$\Delta PE_g = (5.00\times10^{13}\text{ kg})(9.80\text{ m/s}^2)(40.0\text{ m}) = \underline{1.96\times10^{16}\text{ J}}$

(b) From Table 7.1, we know the energy stored in a 9-megaton fusion bomb is $3.8\times10^{16}\text{ J}$, so that $\frac{E_{lake}}{E_{bomb}} = \frac{1.96\times10^{16}\text{ J}}{3.8\times10^{16}\text{ J}} = \underline{0.52}$. The energy stored in the lake is approximately half that of a 9-megaton fusion bomb.

7.7 POWER

30. *The Crab Nebula (see Figure 7.41) pulsar is the remnant of a supernova that occurred in A.D. 1054. Using data from Table 7.3, calculate the approximate factor by which the power output of this astronomical object has declined since its explosion.*

Solution From Table 7.3: $P_{\text{Crab}} = 10^{28}\ \text{W, and}\ P_{\text{Supernova}} = 5\times10^{37}\ \text{W}$ so that $\dfrac{P}{P_0} \approx \dfrac{10^{28} W}{5\times10^{37} W} = \underline{2\times10^{-10}}$. This power today is 10^{10} orders of magnitude smaller than it was at the time of the explosion.

36. *(a) What is the average useful power output of a person who does $6.00\times10^6\ \text{J}$ of useful work in 8.00 h? (b) Working at this rate, how long will it take this person to lift 2000 kg of bricks 1.50 m to a platform? (Work done to lift his body can be omitted because it is not considered useful output here.)*

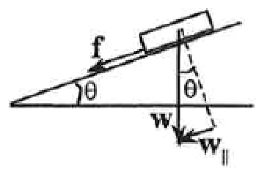
Solution (a) Use $P = \dfrac{W}{t}$ (where *t* is in seconds!):

$$P = \frac{W}{t} = \frac{6.00\times10^6\ \text{J}}{(8.00\ \text{h})(3600\ \text{s}/1\ \text{h})} = 208.3\ \text{J/s} = \underline{208\ \text{W}}$$

(b) Use the work energy theorem to express the work needed to lift the bricks: $W = mgh$, where $m = 2000\ \text{kg}$ and $h = 1.50\ \text{m}$. Then use $P = \dfrac{W}{t}$ to solve for the time: $P = \dfrac{W}{t} = \dfrac{mgh}{t} \Rightarrow t = \dfrac{mgh}{P} = \dfrac{(2000\ \text{kg})(9.80\ \text{m/s}^2)(1.50\ \text{m})}{(208.3\ \text{W})} = 141.1\ \text{s} = \underline{141\ \text{s}}$

42. *Calculate the power output needed for a 950-kg car to climb a 2.00° slope at a constant 30.0 m/s while encountering wind resistance and friction totaling 600 N. Explicitly show how you follow the steps in the Problem-Solving Strategies for Energy.*

Solution The energy supplied by the engine is converted into frictional energy as the car goes up the incline.



$P=\dfrac{W}{t}=\dfrac{Fd}{t}=F\left(\dfrac{d}{t}\right)=Fv$, where F is parallel to the incline and

$F=f+w=600\text{ N}+mg\sin\theta$. Substituting gives $P=(f+mg\sin\theta)v$, so that:

$$P=\left[600\text{ N}+(950\text{ kg})(9.80\text{ m/s}^2)\sin 2°\right](30.0\text{ m/s})=\underline{2.77\times10^4\text{ W}}$$

7.8 WORK, ENERGY, AND POWER IN HUMANS

46. *Calculate the power output in watts and horsepower of a shot-putter who takes 1.20 s to accelerate the 7.27-kg shot from rest to 14.0 m/s, while raising it 0.800 m. (Do not include the power produced to accelerate his body.)*

Solution Use the work energy theorem to determine the work done by the shot-putter:

$$\begin{aligned}\text{net }W&=\frac{1}{2}mv^2+mgh-\frac{1}{2}mv_0^{\,2}-mgh_0\\&=\frac{1}{2}(7.27\text{ kg})(14.0\text{ m/s})^2+(7.27\text{ kg})(9.80\text{ m/s}^2)(0.800\text{ m})=769.5\text{ J}\end{aligned}$$

The power can be found using $P=\dfrac{W}{t}$: $P=\dfrac{W}{t}=\dfrac{769.5\text{ J}}{1.20\text{ s}}=641.2\text{ W}=\underline{641\text{ W}}$.

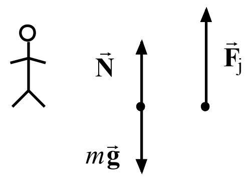
Then, using the conversion 1 hp = 746W, we see that $P=641\text{ W}\times\dfrac{1\text{ hp}}{746\text{ W}}=\underline{0.860\text{ hp}}$

52. *Very large forces are produced in joints when a person jumps from some height to the ground. (a) Calculate the force produced if an 80.0-kg person jumps from a 0.600–m-high ledge and lands stiffly, compressing joint material 1.50 cm as a result. (Be certain to include the weight of the person.) (b) In practice the knees bend almost involuntarily to help extend the distance over which you stop. Calculate the force produced if the stopping distance is 0.300 m. (c) Compare both forces with the weight of the person.*

Solution Given: $m = 80.0\ \text{kg}, h = 0.600\ \text{m}, \text{and}\ d = 0.0150\ \text{m}$

Find: net F. Using $W = Fd$ and the work-energy theorem gives: $W = F_j d = mgh$

$$F = \frac{mgh}{d} = \frac{(80.0\ \text{kg})(9.80\ \text{m/s}^2)(0.600)}{0.0150\ \text{m}} = \underline{3.136\times10^4\ \text{N}}.$$



(a) Now, looking at the body diagram: $\text{net}\ F = w + F_j$

$$\text{net}\ F = (80.0\ \text{kg})(9.80\ \text{m/s}^2) + 3.136\times10^4\ \text{N} = \underline{3.21\times10^4\ \text{N}}$$

(b) Now, let $d = 0.300\ \text{m}$ so that $F_j = \dfrac{mgh}{d} = \dfrac{(80.0\ \text{kg})(9.80\ \text{m/s}^2)(0.600)}{0.300\ \text{m}} = 1568\ \text{N}.$

$$\text{net}\ F = (80.0\ \text{kg})(9.80\ \text{m/s}^2) + 1568\ \text{N} = \underline{2.35\times10^3\ \text{N}}$$

(c) In (a), $\dfrac{\text{net}\ F}{mg} = \dfrac{32{,}144\ \text{N}}{784\ \text{N}} = \underline{41.0}$. This could be damaging to the body.

In (b), $\dfrac{\text{net}\ F}{mg} = \dfrac{2352\ \text{N}}{784\ \text{N}} = \underline{3.00}$. This can be easily sustained.

58. *The awe-inspiring Great Pyramid of Cheops was built more than 4500 years ago. Its square base, originally 230 m on a side, covered 13.1 acres, and it was 146 m high, with a mass of about* $7\times10^9\ \text{kg}$. *(The pyramid's dimensions are slightly different today due to quarrying and some sagging.) Historians estimate that 20,000 workers spent 20 years to construct it, working 12-hour days, 330 days per year. (a) Calculate the gravitational potential energy stored in the pyramid, given its center of mass is at one-fourth its height. (b) Only a fraction of the workers lifted blocks; most were involved in support services such as building ramps (see Figure 7.45), bringing food and water, and hauling blocks to the site. Calculate the efficiency of the workers who did the lifting, assuming there were 1000 of them and they consumed food energy at the rate of 300 kcal/h. What does your answer imply about how much of their work went into block-lifting, versus how much work went into friction and lifting and lowering their own bodies? (c) Calculate the mass of food that had to be supplied each day, assuming that the average worker required 3600 kcal per day and that their diet*

was 5% protein, 60% carbohydrate, and 35% fat. (These proportions neglect the mass of bulk and nondigestible materials consumed.)

Solution (a) To calculate the potential energy use $\text{PE} = mgh$, where $m = 7\times10^9\ \text{kg}$ and $h = \frac{1}{4}\times146\ \text{m} = 36.5\ \text{m}$:

$$\text{PE} = mgh = (7.00\times10^9\ \text{kg})(9.80\ \text{m/s}^2)(36.5\ \text{m}) = 2.504\times10^{12}\ \text{J} = \underline{2.50\times10^{12}\ \text{J}}$$

(b) First, we need to calculate the energy needed to feed the 1000 workers over the 20 years:

$$E_{\text{in}} = NPt = 1000\times\frac{300\ \text{kcal}}{\text{h}}\times\frac{4186\ \text{J}}{\text{kcal}}\times20\ \text{y}\times\frac{330\ \text{d}}{\text{y}}\times\frac{12\ \text{h}}{\text{d}} = 9.946\times10^{13}\ \text{kcal}.$$

Now, since the workers must provide the PE from part (a), use $Eff = \frac{W_{\text{out}}}{E_{\text{in}}}$ to calculate their efficiency: $Eff = \frac{W_{\text{out}}}{E_{\text{in}}} = \frac{\text{PE}}{E_{\text{in}}} = \frac{2.504\times10^{12}\ \text{J}}{9.946\times10^{13}} = 0.0252 = \underline{2.52\%}$

(c) If each worker requires 3600 kcal/day, and we know the composition of their diet, we can calculate the mass of food required:

$$E_{\text{protein}} = (3600\ \text{kcal})(0.05) = 180\ \text{kcal};$$
$$E_{\text{carbohydrate}} = (3600\ \text{kcal})(0.60) = 2160\ \text{kcal; and}$$
$$E_{\text{fat}} = (3600\ \text{kcal})(0.35) = 1260\ \text{kcal}.$$

Now, from Table 7.1 we can convert the energy required into the mass required for each component of their diet:

$$m_{\text{protein}} = E_{\text{protein}}\times\frac{1\ \text{g}}{4.1\ \text{kcal}} = 180\ \text{kcal}\times\frac{1\ \text{g}}{4.1\ \text{kcal}} = 43.90\ \text{g};$$
$$m_{\text{carbohydrate}} = E_{\text{carbohydrate}}\times\frac{1\ \text{g}}{4.1\ \text{kcal}} = 2160\ \text{kcal}\times\frac{1\ \text{g}}{4.1\ \text{kcal}} = 526.8\ \text{g};$$
$$m_{\text{fat}} = E_{\text{fat}}\times\frac{1\ \text{g}}{9.3\ \text{kcal}} = 2160\ \text{kcal}\times\frac{1\ \text{g}}{9.3\ \text{kcal}} = 135.5\ \text{g}.$$

Therefore, the total mass of food require for the average worker per day is:

$$m_{\text{person}} = m_{\text{protein}} + m_{\text{carbohydrate}} + m_{\text{fat}} = (43.90\ \text{g}) + (526.8\ \text{g}) + (135.5\ \text{g}) = 706.2\ \text{g},$$

and the total amount of food required for the 20,000 workers is:

$$m = Nm_{\text{person}} = 20{,}000 \times 0.7062\ \text{kg} = 1.41 \times 10^4\ \text{kg} = \underline{1.4 \times 10^4\ \text{kg}}$$

CHAPTER 8: LINEAR MOMENTUM AND COLLISIONS

8.1 LINEAR MOMENTUM AND FORCE

1. *(a) Calculate the momentum of a 2000-kg elephant charging a hunter at a speed of* $7.50\ \text{m/s}$*. (b) Compare the elephant's momentum with the momentum of a 0.0400-kg tranquilizer dart fired at a speed of* $600\ \text{m/s}$*. (c) What is the momentum of the 90.0-kg hunter running at* $7.40\ \text{m/s}$ *after missing the elephant?*

Solution (a) $p_\text{e} = m_\text{e}v_\text{e} = 2000\ \text{kg} \times 7.50\ \text{m/s} = \underline{1.50 \times 10^4\ \text{kg} \cdot \text{m/s}}$

(b) $p_\text{b} = m_\text{b}v_\text{b} = 0.0400\ \text{kg} \times 600\ \text{m/s} = 24.0\ \text{kg.m/s, so}$

$$\frac{p_\text{c}}{p_\text{b}} = \frac{1.50 \times 10^4\ \text{kg.m/s}}{24.0\ \text{kg.m/s}} = \underline{625}$$

The momentum of the elephant is much larger because the mass of the elephant is much larger.

(c) $p_\text{b} = m_\text{h}v_\text{h} = 90.0\ \text{kg} \times 7.40\ \text{m/s} = \underline{6.66 \times 10^2\ \text{kg} \cdot \text{m/s}}$

Again, the momentum is smaller than that of the elephant because the mass of the hunter is much smaller.

8.2 IMPULSE

9. *A person slaps her leg with her hand, bringing her hand to rest in 2.50 milliseconds from an initial speed of 4.00 m/s. (a) What is the average force exerted on the leg, taking the effective mass of the hand and forearm to be 1.50 kg? (b) Would the force be any different if the woman clapped her hands together at the same speed and brought them to rest in the same time? Explain why or why not.*

Solution (a) Calculate the net force on the hand:

$$\text{net } F = \frac{\Delta p}{\Delta t} = \frac{m\Delta v}{\Delta t} = \frac{1.50\text{kg}(0\text{ m/s} - 4.00\text{ m/s})}{2.50 \times 10^{-3}\text{ s}} = \underline{-2.40 \times 10^{3}\text{ N}}$$

(taking moment toward the leg as positive). Therefore, by Newton's third law, the net force exerted on the leg is $\underline{2.40 \times 10^{3}\text{ N}}$, toward the leg.

(b) The force on each hand would have the same magnitude as that found in part (a) (but in opposite directions by Newton's third law) because the changes in momentum and time interval are the same.

15. *A cruise ship with a mass of* $1.00 \times 10^{7}\text{ kg}$ *strikes a pier at a speed of 0.750 m/s. It comes to rest 6.00 m later, damaging the ship, the pier, and the tugboat captain's finances. Calculate the average force exerted on the pier using the concept of impulse. (Hint: First calculate the time it took to bring the ship to rest.)*

Solution Given: $m = 1.00 \times 10^{7}\text{ kg},\ v_0 = 0.75\text{ m/s}, v = 0\text{ m/s}, \Delta x = 6.00\text{ m}.$ Find: net force on the pier. First, we need a way to express the time, Δt, in terms of known quantities. Using the equations $\bar{v} = \frac{\Delta x}{\Delta t}$ and $\bar{v} = \frac{v_0 + v}{2}$ gives:

$$\Delta x = \bar{v}\Delta t = \frac{1}{2}(v + v_0)\Delta t \text{ so that } \Delta t = \frac{2\Delta x}{v + v_0} = \frac{2(6.00\text{ m})}{(0 + 0.750)\text{m/s}} = \underline{16.0\text{ s.}}$$

$$\text{net } F = \frac{\Delta p}{\Delta t} = \frac{m(v - v_0)}{\Delta t} = \frac{(1.00 \times 10^{7}\text{ kg})(0 - 0750)\text{m/s}}{16.0\text{ s}} = \underline{-4.69 \times 10^{5}\text{ N.}}$$

By Newton's third law, the net force on the pier is $\underline{4.69 \times 10^{5}\text{ N}}$, in the original direction of the ship.

8.3 CONSERVATION OF MOMENTUM

23. ***Professional Application*** *Train cars are coupled together by being bumped into one another. Suppose two loaded train cars are moving toward one another, the first having a mass of 150,000 kg and a velocity of 0.300 m/s, and the second having a mass of 110,000 kg and a velocity of* -0.120 m/s*. (The minus indicates direction of motion.) What is their final velocity?*

Solution Use conservation of momentum, $m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2'$, since their final velocities are the same.

$$v' = \frac{m_1v_1 + m_2v_2}{m_1 + m_2} = \frac{(150{,}000\ \text{kg})(0.300\ \text{m/s}) + (110{,}000\ \text{kg})(-0.120\ \text{m/s})}{150{,}000\ \text{kg} + 110{,}000\ \text{kg}} = \underline{0.122\ \text{m/s}}$$

The final velocity is in the direction of the first car because it had a larger initial momentum.

8.5 INELASTIC COLLISIONS IN ONE DIMENSION

33. ***Professional Application*** *Using mass and speed data from Example 8.1 and assuming that the football player catches the ball with his feet off the ground with both of them moving horizontally, calculate: (a) the final velocity if the ball and player are going in the same direction and (b) the loss of kinetic energy in this case. (c) Repeat parts (a) and (b) for the situation in which the ball and the player are going in opposite directions. Might the loss of kinetic energy be related to how much it hurts to catch the pass?*

Solution (a) Use conservation of momentum for the player and the ball:
$m_1v_1 + m_2v_2 = (m_1 + m_2)v'$ so that

$$v' = \frac{m_1v_1 + m_2v_2}{m_1 + m_2} = \frac{(110\ \text{kg})(8.00\ \text{m/s}) + (0.410\ \text{kg})(25.0\ \text{m/s})}{110\ \text{kg} + 0.410\ \text{kg}} = 8.063\ \text{m/s} = \underline{8.06\ \text{m/s}}$$

(b) $\Delta\text{KE} = \text{KE}' - (\text{KE}_1 + \text{KE}_2)$

$$= \frac{1}{2}m_1v_1'^2 + \frac{1}{2}m_2v_2'^2 - \frac{1}{2}m_1v_1^2 - \frac{1}{2}m_2v_2^2 = \frac{1}{2}(m_1 + m_2)v'^2 - \frac{1}{2}(m_1v_1'^2 + m_2v_2'^2)$$

$$= \frac{1}{2}(110.41\ \text{kg})(8.063\ \text{m/s})^2 - \frac{1}{2}\left[(110\ \text{kg})(8.00\ \text{m/s})^2 + (0.400\ \text{kg})(25.0\ \text{m/s})^2\right]$$

$$= \underline{-59.0\ \text{J}}$$

(c) (i) $v' = \dfrac{(110\ \text{kg})(8.00\ \text{m/s}) + (0.410\ \text{kg})(-25.0\ \text{m/s})}{110.41\ \text{kg}} = \underline{7.88\ \text{m/s}}$

(ii) $\Delta\text{KE} = \frac{1}{2}(110.41\,\text{kg})(7.877\,\text{m/s})^2 -$

$$\frac{1}{2}\left[(110\,\text{kg})(8.00\,\text{m/s})^2 + \frac{1}{2}(0.410\,\text{kg})(-25.0\,\text{m/s})^2\right] = \underline{-223\,\text{J}}$$

38. *A 0.0250-kg bullet is accelerated from rest to a speed of 550 m/s in a 3.00-kg rifle. The pain of the rifle's kick is much worse if you hold the gun loosely a few centimeters from your shoulder rather than holding it tightly against your shoulder. (a) Calculate the recoil velocity of the rifle if it is held loosely away from the shoulder. (b) How much kinetic energy does the rifle gain? (c) What is the recoil velocity if the rifle is held tightly against the shoulder, making the effective mass 28.0 kg? (d) How much kinetic energy is transferred to the rifle-shoulder combination? The pain is related to the amount of kinetic energy, which is significantly less in this latter situation. (e) See Example 8.1 and discuss its relationship to this problem.*

Solution (a) Given: $v_1 = v_2 = 0\,\text{m/s},\ m_1 = 3.00\,\text{kg}$. Use conservation of momentum:

$$m_1 v_1 + m_2 v_2 = (m_1 + m_2)v'$$

$$m_1 v'_1 = -m_2 v'_2 \Rightarrow v'_1 = \frac{-m_2 v'_2}{m_1} = \frac{-(0.0250\,\text{kg})(550\,\text{m/s})}{3.00\,\text{kg}} = -4.583\,\text{m/s} = \underline{4.58\,\text{m/s}}$$

(b) The rifle begins at rest, so $\text{KE}_\text{i} = 0\,\text{J}$, and

$$\Delta\text{KE} = \frac{1}{2}m_1 v'^2_1 = \frac{1}{2}(3.00\,\text{kg})(-4.58\,\text{m/s})^2 = \underline{31.5\,\text{J}}$$

(c) Now, $m_1 = 28.0\,\text{kg}$, so that $v'_1 = \frac{-m_2 v_2}{m_1} = \frac{-(0.0250\,\text{kg})(550\,\text{m/s})}{28.0\,\text{kg}} = \underline{-0.491\,\text{m/s}}$

(d) Again, $\text{KE}_\text{i} = 0\,\text{J}$, and

$$\Delta\text{KE} = \frac{1}{2}m_1 v'^2_1 = \frac{1}{2}(28.0\,\text{kg})(-0.491\,\text{m/s})^2 = 3.376\,\text{J} = \underline{3.38\,\text{J}}$$

(e) Example 8.1 makes the observation that if two objects have the same momentum the heavier object will have a smaller kinetic energy. Keeping the rifle close to the body increases the effective mass of the rifle, hence reducing the kinetic energy of the recoiling rifle. Since pain is related to the amount of kinetic energy, a rifle hurts less if it held against the body.

44. *(a) During an ice skating performance, an initially motionless 80.0-kg clown throws a fake barbell away. The clown's ice skates allow her to recoil frictionlessly. If the clown recoils with a velocity of 0.500 m/s and the barbell is thrown with a velocity of 10.0 m/s, what is the mass of the barbell? (b) How much kinetic energy is gained by this maneuver? (c) Where does the kinetic energy come from?*

Solution (a) Use conversation of momentum to find the mass of the barbell:
$m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2'$ where $v_1 = v_2 = 0\ \text{m/s}$, and v_1'=-0.500 m/s (since it recoils backwards), so solving for the mass of the barbell gives:

$$0 = m_1v_1 + m_2v_2 \Rightarrow m_2 = \frac{-m_1v_1}{v_2} = \frac{-(80.0\ \text{kg})(-0.500\ \text{m/s})}{10.0\ \text{m/s}} = \underline{4.00 \text{kg}}$$

(b) Find the change in kinetic energy:

$$\Delta \text{KE} = \frac{1}{2}m_1v'^2_1 + \frac{1}{2}m_2v'^2_2 - \frac{1}{2}m_1v_1^2 - \frac{1}{2}m_2v_2^2 = \frac{1}{2}\left(m_1v'^2_1 + m_2v'^2_2\right)$$
$$= \frac{1}{2}\left[(80.0\ \text{kg})(-0.500\ \text{m/s})^2 + \frac{1}{2}(4.00\ \text{kg})(10.0\ \text{m/s})^2\right] = \underline{210\ \text{J}}$$

(c) The clown does work to throw the barbell, so the kinetic energy comes from the muscles of the clown. The muscles convert the chemical potential energy of ATP into kinetic energy.

8.6 COLLISIONS OF POINT MASSES IN TWO DIMENSIONS

49. ***Professional Application*** *Ernest Rutherford (the first New Zealander to be awarded the Nobel Prize in Chemistry) demonstrated that nuclei were very small and dense by scattering helium-4 nuclei* (^{4}He) *from gold-197 nuclei* (^{197}Au). *The energy of the incoming helium nucleus was* $8.00\times10^{-13}\ \text{J}$, *and the masses of the helium and gold nuclei were* $6.68\times10^{-27}\ \text{kg}$ *and* $3.29\times10^{-25}\ \text{kg}$, *respectively (note that their mass ratio is 4 to 197). (a) If a helium nucleus scatters to an angle of* $120°$ *during an elastic collision with a gold nucleus, calculate the helium nucleus's final speed and the final velocity (magnitude and direction) of the gold nucleus. (b) What is the final kinetic energy of the helium nucleus?*

Solution (a) $\frac{1}{2}m_1 v_1^2 = \text{KE}_\text{i} \Rightarrow v_\text{i} = \left(\frac{2\text{KE}_\text{i}}{m_1}\right)^{1/2} = \left[\frac{2(8.00\times10^{-13}\text{ J})}{6.68\times10^{-27}\text{ kg}}\right]^{1/2} = 1.548\times10^7\text{ m/s}$

Conservation of internal kinetic energy gives:

$$\frac{1}{2}m_1 {v_1}^2 = \frac{1}{2}m_1 v'^2_1 + \frac{1}{2}m_1 v'^2_2 \qquad \text{(i)}$$

$$\text{or} \quad \frac{m_1}{m_2}\left(v'^2_1 - v'^2_{2_1'}\right) = v'^2_2 \qquad \text{(i')}$$

Conservation of momentum along the *x*-axis gives:

$$m_1 v_1 = m_1 v'_1 \cos\theta_1 + m_2 v'_2 \cos\theta_2 \qquad \text{(ii)}$$

Conservation of momentum along the *y*-axis gives:

$$0 = m_1 v'_1 \sin\theta_1 + m_2 v'_2 \sin\theta_2 \qquad \text{(iii)}$$

Rearranging Equations (ii) and (iii) gives:

$$m_1 v_1 - m_1 v'_1 \cos\theta_1 = m_2 v'_2 \cos\theta_2 \qquad \text{(ii')}$$

$$-m_1 v'_1 \sin\theta_1 = m_2 v'_2 \sin\theta_2 \qquad \text{(iii')}$$

Squaring Equation (ii') and (iii') and adding gives:

$$m_2^2 v'^2_2 \cos^2\theta_2 + m_2^2 v'^2_2 \sin^2\theta_2 = (m_1 v_1 - m_1 v'_1 \cos\theta_1)^2 + \left(-m_1 v'_1 \cos\theta_1\right)^2$$
$$\text{or} \quad m_2^2 v'^2_2 = m_1^2 v'^2_1 - 2m_1^2 v_1 v'_1 \cos\theta_1 + m_1^2 v'^2_1$$

Solving for v'^2_2 and substituting into (i'):

$$\frac{m_1}{m_2}\left(v_1^2 - v'^2_1\right) = \frac{m_1^2}{m_2^2}\left(v_1^2 + v'^2_1 - 2v_1 v'_1 \cos\theta_1\right) \text{so that}$$

$$v_1^2 - v'^2_1 = \frac{m_1}{m_2}\left(v_1^2 + v'^2_1 - 2v_1 v'_1 \cos\theta_1\right)$$

Using $v_1 = 1.548\times10^7\ \text{m/s}; \theta_1 = 120°; m_1 = 6.68\times10^{-27}\ \text{kg}; m_2 = 3.29\times10^{-25}\ \text{kg}$

$$\left(1+\frac{m_1}{m_2}\right)v'^2_1 - \left(2\frac{m_1}{m_2}v_1\cos\theta_1\right)v'_1 - \left(1-\frac{m_1}{m_2}\right)v_1^2 = 0$$

$$a = 1+\frac{m_1}{m_2} = 1.0203,\ b = -\frac{2m_1}{m_2}v_1\cos\theta_1 = 3.143\times10^5\ \text{m/s},$$

$$c = -\left(1+\frac{m_1}{m_2}\right)v_1^2 = -2.348\times10^{14}\ \text{m}^2/\text{s}^2 \text{ so that}$$

$$v'_1 = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$= \frac{-3.143\times10^5\ \text{m/s} + \sqrt{\left(3.143\times10^5\ \text{m/s}\right)^2 - 4(1.0203)\left(-2.348\times10^{14}\ \text{m}^2/\text{s}^2\right)}}{2(1.0203)} \text{ or}$$

$$v'_1 = \underline{1.50\times10^7\ \text{m/s}} \text{ and } v'_2 = \sqrt{\frac{m_1}{m_2}\left(v_1^2 - v'_1\right)} = \underline{5.36\times10^5\ \text{m/s}}$$

$$\tan\theta_2 = \frac{-v'_1\sin\theta_1}{v_1 - v'_1\cos\theta_1} = \frac{=\left(1.50\times10^7\ \text{m/s}\right)\sin120°}{1.58\times10^7\ \text{m/s} - \left(1.50\times10^7\ \text{m/s}\right)\cos120°} = -0.56529$$

$$\text{or } \theta_2 = \tan^{-1}(-0.56529) = \underline{-29.5°}$$

(b) The final kinetic energy is then:

$$\text{KE}_\text{f} = (0.5)m_1v'^2_1 = (0.5)\left(6.68\times10^{-27}\ \text{kg}\right)\left(1.50\times10^7\ \text{m/s}\right)^2 = \underline{7.52\times10^{-13}\ \text{J}}$$

8.7 INTRODUCTION TO ROCKET PROPULSION

55. ***Professional Application*** *Calculate the increase in velocity of a 4000-kg space probe that expels 3500 kg of its mass at an exhaust velocity of* $2.00\times10^3\ \text{m/s}$*. You may assume the gravitational force is negligible at the probe's location.*

Solution

Use the equation $v = v_0 + v_\text{e}\ln\left(\frac{m_0}{m}\right)$, where

$m_0 = 4000\ \text{kg},\ m = 4000\ \text{kg - }3500\ \text{kg} = 500\ \text{kg, and } v_\text{e} = 2.00\times10^3\ \text{m/s}$ so that

$$v - v_0 = (2.00\times10^3\ \text{m/s})\ln\left(\frac{4000\ \text{kg}}{500\ \text{kg}}\right) = 4.159\times10^3\ \text{m/s} = \underline{4.16\times10^3\ \text{m/s}}$$

57. *Derive the equation for the vertical acceleration of a rocket.*

Solution The force needed to give a small mass Δm an acceleration $a_{\Delta m}$ is $F = \Delta m a_{\Delta m}$. To accelerate this mass in the small time interval Δt at a speed v_e requires $v_\text{e} = a_{\Delta m}\Delta t$, so $F = v_\text{e}\dfrac{\Delta m}{\Delta t}$. By Newton's third law, this force is equal in magnitude to the thrust force acting on the rocket, so $F_\text{thrust} = v_e\dfrac{\Delta m}{\Delta t}$, where all quantities are positive. Applying Newton's second law to the rocket gives $F_\text{thrust} - mg = ma \Rightarrow a = \dfrac{v_e}{m}\dfrac{\Delta m}{\Delta t} - g$, where m is the mass of the rocket and unburnt fuel.

61. ***Professional Application*** *(a) A 5.00-kg squid initially at rest ejects 0.250-kg of fluid with a velocity of 10.0 m/s. What is the recoil velocity of the squid if the ejection is done in 0.100 s and there is a 5.00-N frictional force opposing the squid's movement. (b) How much energy is lost to work done against friction?*

Solution (a) First, find v'_1, the velocity after ejecting the fluid:

$$(m_1 + m_2)v = 0 = m_1v'_1 + m_2v'_2\text{, so that}$$

$$v'_1 = \frac{-m_2v'_2}{m_1} = \frac{-(0.250\ \text{kg})(10.0\ \text{m/s})}{4.75\ \text{kg}} = \underline{-0.526\ \text{m/s}}$$

Now, the frictional force slows the squid over the 0.100 s

$$\Delta p = f\Delta t = m_1v'_{1,\text{f}} + m_2v'_2,\ \text{gives:}$$

$$v'_{1,\text{f}} = \frac{f\Delta t - m_2v'_2}{m_1} = \frac{(5.00\ \text{N})(0.100\ \text{s}) - (0.250\ \text{kg})(10.0\ \text{m/s})}{4.75\ \text{kg}} = \underline{-0.421\ \text{m/s}}$$

(b) $$\Delta\text{KE} = \frac{1}{2}m_1v'^2_{1,\text{f}} - \frac{1}{2}m_1v'^2_1 = \frac{1}{2}m_1\left(v'^2_{1,\text{f}} - v'^2_1\right)$$

$$= \frac{1}{2}(4.75\ \text{kg})\left[(0.421\ \text{m/s})^2 - (0.526\ \text{m/s})^2\right] = \underline{-0.236\ \text{J}}$$

CHAPTER 9: STATICS AND TORQUE

9.2 THE SECOND CONDITION FOR EQUILIBRIUM

1. *(a) When opening a door, you push on it perpendicularly with a force of 55.0 N at a distance of 0.850m from the hinges. What torque are you exerting relative to the hinges? (b) Does it matter if you push at the same height as the hinges?*

Solution (a) To calculate the torque use $\tau = r_\perp F$, where the perpendicular distance is 0.850 m, the force is 55.0 N, and the hinges are the pivot point.

$$\tau = r_\perp F = 0.850\,\text{m} \times 55.0\,\text{N} = 46.75\,\text{N}\cdot\text{m} = \underline{46.8\,\text{N}\cdot\text{m}}$$

(b) It does not matter at what height you push. The torque depends on only the magnitude of the force applied and the perpendicular distance of the force's application from the hinges. (Children don't have a tougher time opening a door because they push lower than adults, they have a tougher time because they don't push far enough from the hinges.)

9.3 STABILITY

6. *Suspose a horse leans against a wall as in Figure 9.32. Calculate the force exerted on the wall assuming that force is horizontal while using the data in the schematic representation of the situation. Note that the force exerted on the wall is equal and opposite to the force exerted on the horse, keeping it in equilibrium. The total mass of the horse and rider is 500 kg. Take the data to be accurate to three digits.*

Solution There are four forces acting on the horse and rider: **N** (acting straight up the ground), **w** (acting straight down from the center of mass), **f** (acting horizontally to the left, at the ground to prevent the horse from slipping), and $\mathbf{F}_{\text{wall}}$ (acting to the right). Since nothing is moving, the two conditions for equilibrium apply: $\text{net } F = 0$ and $\text{net } \tau = 0$.

The first condition leads to two equations (one for each direction):

$$\text{net } F_x = F_{\text{wall}} - f = 0 \text{ and } \text{net } F_y = N - w = 0$$

The torque equation (taking torque about the center of gravity, where CCW is positive) gives: $\text{net } \tau = F_{\text{wall}}(1.40 - 1.20) - f(1.40\text{ m}) + N(0.350\text{ m}) = 0$

The first two equations give: $F_{\text{wall}} = f$, and $N = w = mg$

Substituting into the third equation gives:

$$F_{\text{wall}}(1.40\text{ m} - 1.20\text{ m}) - F_{\text{wall}}(1.40\text{ m}) = -mg(0.350\text{ m})$$

So, the force on the wall is:

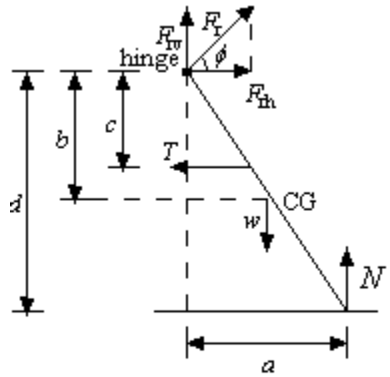
$$F_{\text{wall}} = \frac{mg(0.350\text{ m})}{1.20\text{ m}} = \frac{(500\text{ kg})(9.80\text{ m/s}^2)(0.350\text{ m})}{1.20\text{ m}} = 1429\text{ N} = \underline{1.43 \times 10^3\text{ N}}$$

14. *A sandwich board advertising sign is constructed as shown in Figure 9.36. The sign's mass is 8.00 kg. (a) Calculate the tension in the chain assuming no friction between the legs and the sidewalk. (b) What force is exerted by each side on the hinge?*

Solution Looking at Figure 9.36, there are three forces acting on the entire sandwich board system: $\mathbf{w}$, acting down at the center of mass of the system, $\mathbf{N}_\text{L}$ and $\mathbf{N}_\text{R}$, acting up at the ground for EACH of the legs. The tension and the hinge exert internal forces, and therefore cancel when considering the entire sandwich board. Using the first condition for equilibrium gives: $\text{net } F = N_\text{L} + N_\text{R} - w_\text{S}$.

The normal forces are equal, due to symmetry, and the mass is given, so we can determine the normal forces: $2N = mg = (8.00\text{ kg})(9.80\text{ m/s}^2) \Rightarrow N = 39.2N$

Now, we can determine the tension in the chain and the force due to the hinge by using the one side of the sandwich board:



$$a = \frac{1.10}{2} = 0.550\text{ m}, b = \frac{1.30\text{ m}}{2} = 0.650\text{ m}, c = 0.500\text{ m}, d = 1.30\text{ m},$$

$$N = 39.2\text{ N}, w = mg = \frac{8.00\text{ kg}}{2}\left(9.80\text{ m/s}^2\right) = 39.2\text{ N (for one side)}$$

$$F_{rv} = F_r \sin\phi, F_{rh} = F_r \cos\phi$$

The system is in equilibrium, so the two conditions for equilibrium hold:

$$\text{net } F = 0 \text{ and net } \tau = 0$$

This gives three equations:

$$\text{net } F_x = F_{rh} - T = 0$$
$$\text{net } F_y = F_{rv} - w + N = 0$$
$$\text{net } \tau = -Tc - w\frac{a}{2} + Na = 0$$

(Pivot at hinge)

$$\text{Giving } F_{rh} = T = F_r \cos\phi,$$
$$F_{rv} = F_r \sin\phi, F_{rh} = F_r \cos\phi$$
$$F_{rv} = w - N = F_r \sin\phi, \text{ and } Tc + w\frac{a}{2} = Na$$

(a) To solve for the tension, use the third equation:

$$Tc = Na - w\frac{a}{2} \Rightarrow T = \frac{Na}{c} - \frac{wa}{2c} = \frac{wa}{2c}$$

Since $N = w$

Therefore, substituting in the values gives: $T = \dfrac{(39.2\text{ N})(0.550\text{ m})}{2(0.500\text{ m})} = \underline{21.6\text{ N}}$

(b) To determine the force of the hinge, and the angle at which it acts, start with the second equation, remembering that $N = w, F_{\text{rv}} = w - N \Rightarrow F_{\text{rv}} = 0.$

Now, the first equation says: $F_{\text{rh}} = T$, so F_{r} cannot be zero, but rather $\phi = 0$, giving a force of $F_r = \underline{21.6\text{ N (acting horizontally)}}$

9.6 FORCES AND TORQUES IN MUSCLES AND JOINTS

32. *Even when the head is held erect, as in Figure 9.42, its center of mass is not directly over the principal point of support (the atlanto-occipital joint). The muscles at the back of the neck should therefore exert a force to keep the head erect. That is why your head falls forward when you fall asleep in the class. (a) Calculate the force exerted by these muscles using the information in the figure. (b) What is the force exerted by the pivot on the head?*

Solution (a) Use the second condition for equilibrium:

$$\text{net }\tau = F_{\text{M}}(0.050\text{ m}) - w(0.025\text{ m}) = 0, \text{ so that}$$

$$F_{\text{M}} = w\frac{0.025\text{ m}}{0.050\text{ m}} = (50\text{ N})\frac{0.025\text{ m}}{0.050\text{ m}} = \underline{25\text{ N downward}}$$

(b) To calculate the force on the joint, use the first condition of equilibrium:

$$\text{net }F_y = F_{\text{J}} - F_{\text{M}} - w = 0, \text{ so that}$$

$$F_{\text{J}} = F_{\text{M}} + w = (25\text{ N}) + (50\text{ N}) = \underline{75\text{ N upward}}$$

CHAPTER 10: ROTATIONAL MOTION AND ANGULAR MOMENTUM

10.1 ANGULAR ACCELERATION

1. *At its peak, a tornado is 60.0 m in diameter and carries 500 km/h winds. What is its angular velocity in revolutions per second?*

Solution First, convert the speed to m/s: $v = \frac{500\text{ km}}{1\text{ h}} \times \frac{1\text{ h}}{3600\text{ s}} \times \frac{1000\text{ m}}{1\text{ km}} = 138.9\text{ m/s}.$

Then, use the equation $\omega = \frac{v}{r}$ to determine the angular speed:

$$\omega = \frac{v}{r} = \frac{138.9\text{ m/s}}{30.0\text{ m}} = 4.630\text{ rad/s}.$$

Finally, convert the angular speed to rev/s: $\omega = 4.630\text{ rad/s} \times \frac{1\text{ rev}}{2\pi\text{ rad}} = \underline{0.737\text{ rev/s}}$

3. ***Integrated Concepts*** *You have a grindstone (a disk) that is 90.0 kg, has a 0.340-m radius, and is turning at 90.0 rpm, and you press a steel axe against it with a radial force of 20.0 N. (a) Assuming the kinetic coefficient of friction between steel and stone is 0.20, calculate the angular acceleration of the grindstone. (b) How many turns will the stone make before coming to rest?*

Solution (a) Given: $M = 90.0\text{ kg}$, $R = 0.340\text{ m}$ (for the solid disk), $\omega = 90.0\text{ rev/min}$, $N = 20.0\text{ N}$, and $\mu_\text{k} = 0.20.$

Find α. The frictional force is given by $f = \mu_\text{k} N = (0.20)(20.0\text{ N}) = 4.0\text{ N}$. This frictional force is reducing the speed of the grindstone, so the angular acceleration will be negative. Using the moment of inertia for a solid disk and $\tau = I\alpha$ we know: $\tau = -fR = I\alpha = \frac{1}{2}MR^2\alpha$. Solving for angular acceleration gives:

$$\alpha = \frac{-2f}{MR} = \frac{-2(4.0\ \text{N})}{(90.0\ \text{kg})(0.340\ \text{m})}$$
$$= -0.261\ \text{rad/s}^2 = \underline{-0.26\ \text{rad/s}^2}(2\ \text{sig. figs due to}\ \mu_\text{k}).$$

(b) Given: $\omega = 0\ \text{rad/s},\ \omega_0 = \frac{90.0\ \text{rev}}{\text{min}} \times \frac{2\pi\ \text{rad}}{\text{rev}} \times \frac{1\ \text{min}}{60\ \text{s}} = 9.425\ \text{rad/s}.$

Find θ. Use the equation $\omega^2 - \omega_0^2 = 2\alpha\theta$, so that:

$$\theta = \frac{\omega^2 - \omega_0^2}{2\alpha} = \frac{(0\ \text{rad/s})^2 - (9.425\ \text{rad/s})^2}{2(-0.261\ \text{rad/s}^2)}$$
$$= 170.2\ \text{rad} \times \frac{1\ \text{rev}}{2\pi\ \text{rad}} = 27.0\ \text{rev} = \underline{27\ \text{rev}}.$$

10.3 DYNAMICS OF ROTATIONAL MOTION: ROTATIONAL INERTIA

10. *This problem considers additional aspects of example Calculating the Effect of Mass Distribution on a Merry-Go-Round. (a) How long does it take the father to give the merry-go-round and child an angular velocity of 1.50 rad/s? (b) How many revolutions must he go through to generate this velocity? (c) If he exerts a slowing force of 300 N at a radius of 1.35 m, how long would it take him to stop them?*

Solution (a) Using the result from Example 10.7:

$$\alpha = 4.44\ \text{rad/s}^2, \omega_0 = 0.00\ \text{rad/s, and}\ \omega = 1.50\ \text{rad/s},$$

we can solve for time using the equation, $\omega = \omega_0 + \alpha t$, or

$$t = \frac{\omega - \omega_0}{\alpha} = \frac{(1.50\ \text{rad/s}) - (0\ \text{rad/s})}{4.44\ \text{rad/s}^2} = \underline{0.338\ \text{s}}.$$

(b) Now, to find θ without using our result from part (a), use the equation

$\omega^2 = \omega_0^2 + 2\alpha\theta$, giving:

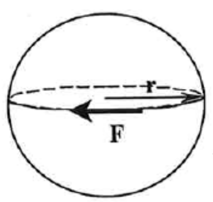
$$\theta = \frac{\omega^2 - \omega_0^2}{2\alpha} = \frac{(1.50\text{ rad/s})^2 - (0\text{ rad/s})^2}{2(4.44\text{ rad/s}^2)} = 0.253\text{ rad} \times \frac{1\text{ rev}}{2\pi\text{ rad}} = \underline{0.0403\text{ rev}}$$

(c) To get an expression for the angular acceleration, use the equation $\alpha = \frac{\text{net }\tau}{I} = \frac{rF}{I}$. Then, to find time, use the equation:

$$t = \frac{\omega - \omega_0}{\alpha} = \frac{(\omega - \omega_0)I}{rF} = \frac{(0\text{ rad/s} - 1.50\text{ rad/s})(84.38\text{ kg.m}^2)}{(1.35\text{ m})(-300\text{ N})} = \underline{0.313\text{ s}}$$

16. *Zorch, an archenemy of Superman, decides to slow Earth's rotation to once per 28.0 h by exerting an opposing force at and parallel to the equator. Superman is not immediately concerned, because he knows Zorch can only exert a force of* $4.00\times10^7\text{ N}$ *(a little greater than a Saturn V rocket's thrust). How long must Zorch push with this force to accomplish his goal? (This period gives Superman time to devote to other villains.) Explicitly show how you follow the steps found in Problem-Solving Strategy for Rotational Dynamics.*

Solution



Step 1: There is a torque present due to a force being applied perpendicular to a rotation axis. The mass involved is the earth.

Step 2: The system of interest is the earth.

Step 3: The free body diagram is drawn to the left.

Step 4: Given:

$$F = -4.00\times10^{7}\ \text{N}, r = r_{\text{E}} = 6.376\times10^{6}\ \text{m}, M = 5.979\times10^{24}\ \text{kg},$$

$$\omega_0 = \frac{1\ \text{rev}}{24.0\ \text{h}}\times\frac{2\pi\ \text{rad}}{1\ \text{rev}}\times\frac{1\ \text{h}}{3600\ \text{s}} = 7.272\times10^{-5}\ \text{rad/s, and}$$

$$\omega = \frac{1\ \text{rev}}{28.0\ \text{h}}\times\frac{2\pi\ \text{rad}}{1\ \text{rev}}\times\frac{1\ \text{h}}{3600\ \text{s}} = 6.233\times10^{-5}\ \text{rad/s.}$$

Find t.

Use the equation $\alpha = \frac{\text{net}\ \tau}{I}$ to determine the angular acceleration:

$$\alpha = \frac{\text{net}\ \tau}{I} = \frac{rF}{2Mr^2/5} = \frac{5F}{2Mr}$$

Now that we have an expression for the angular acceleration, we can use the equation $\omega = \omega_0 + \alpha t$ to get the time:

$$\omega = \omega_0 + \alpha t,\ \Rightarrow t = \frac{\omega - \omega_0}{\alpha} = \frac{(\omega - \omega_0)2Mr}{5F}$$

Substituting in the number gives:

$$t = \frac{2(6.233\times10^{-5}\ \text{rad/s} - 7.272\times10^{-5}\ \text{rad/s})(5.979\times10^{24}\ \text{kg})(6.376\times10^{6}\ \text{m})}{5(-4.00\times10^{7}\ \text{N})}.$$

$$= \underline{3.96\times10^{18}\ \text{s}}\ \text{or}\ \underline{1.25\times10^{11}\ \text{y}}$$

10.4 ROTATIONAL KINETIC ENERGY: WORK AND ENERGY REVISITED

24. *Calculate the rotational kinetic energy in the motorcycle wheel (Figure 10.38) if its angular velocity is 120 rad/s.*

Solution The moment of inertia for the wheel is

$$I = \frac{M}{2}\left(R_1^2 + R_2^2\right) = \frac{12.0\text{ kg}}{2}\left[(0.280\text{ m})^2 + (0.330\text{ m})^2\right] = 1.124\text{ kg}\cdot\text{m}^2$$

Using the equation: $\text{KE}_{\text{rot}} = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(1.124\text{ kg}\cdot\text{m}^2\right)(120\text{ rad/s})^2 = \underline{8.09\times10^3\text{ J}}$

30. *To develop muscle tone, a woman lifts a 2.00-kg weight held in her hand. She uses her biceps muscle to flex the lower arm through an angle of* $60.0°$*. (a) What is the angular acceleration if the weight is 24.0 cm from the elbow joint, her forearm has a moment of inertia of* $0.250\text{ kg}\cdot\text{m}^2$*, and the muscle force is 750 N at an effective perpendicular lever arm of 2.00 cm? (b) How much work does she do?*

Solution (a) Assuming her arm starts extended vertically downward, we can calculate the initial angular acceleration.

$$\text{Given}: \theta = 60°\times\frac{2\pi\text{ rad}}{360°} = 1.047\text{ rad, m}_\text{w} = 2.00\text{ kg},\ r_\text{w} = 0.240\text{ m},$$
$$I = 0.250\text{ kg.m}^2\text{, and } F = 750\text{ N, where } r_\perp = 0.0200\text{ m}.$$

Find α.

The only force that contributes to the torque when the mass is vertical is the muscle, and the moment of inertia is that of the arm and that of the mass. Therefore:

$$\alpha = \frac{\text{net }\tau}{I + m_\text{w}r_\text{w}^2} = \frac{F_\text{M}r_\perp}{I + m_\text{w}r_\text{w}^2}\text{ so that}$$
$$\alpha = \frac{(750\text{ N})(0.0200\text{ m})}{0.250\text{ kg}\cdot\text{m}^2 + (2.00\text{ kg})(0.240\text{ m})^2} = 41.07\text{ rad/s}^2 = \underline{41.1\text{ rad/s}^2}$$

(b) The work done is:

$$\text{net }W = (\text{net }\tau)\theta = F_\text{M}r_\perp\theta = (750\text{ N})(0.0200\text{ m})(1.047\text{ rad}) = \underline{15.7\text{ J}}$$

10.5 ANGULAR MOMENTUM AND ITS CONSERVATION

36. *(a) Calculate the angular momentum of the Earth in its orbit around the Sun. (b) Compare this angular momentum with the angular momentum of Earth on its axis.*

Solution (a) The moment of inertia for the earth around the sun is $I = MR^2$, since the earth is like a point object.

$$I = MR^2,$$
$$L_{\text{orb}} = I\omega = MR^2\omega$$
$$= (5.979\times10^{24}\text{ kg})(1.496\times10^{11}\text{ m})^2\left(\frac{2\pi\text{ rad}}{3.16\times10^7\text{ s}}\right) = \underline{2.66\times10^{40}\text{ kg}\cdot\text{m}^2/\text{s}}$$

(b) The moment of inertia for the earth on its axis is $I = \dfrac{2MR^2}{5}$, since the earth is a solid sphere.

$$I = \frac{2MR^2}{5},$$
$$L_{\text{orb}} = \left(\frac{2}{5}MR^2\right)\omega = \frac{2}{5}(5.979\times10^{24}\text{ kg})(6.376\times10^6\text{ m})^2\left(\frac{2\pi\text{ rad}}{24\times3600\text{ s}}\right)$$
$$= \underline{7.07\times10^{33}\text{ kg}\cdot\text{m}^2/\text{s}}$$

The angular momentum of the earth in its orbit around the sun is 3.76×10^6 times larger than the angular momentum of the earth around its axis.

10.6 COLLISIONS OF EXTENDED BODIES IN TWO DIMENSIONS

43. *Repeat Example 10.15 in which the disk strikes and adheres to the stick 0.100 m from the nail.*

Solution (a) The final moment of inertia is again the disk plus the stick, but this time, the radius for the disk is smaller:

$$I' = mr^2 + \frac{MR^2}{3} = (0.0500\ \text{kg})(0.100\ \text{m})^2 + (0.667\ \text{kg})(1.20\ \text{m})^2 = 0.961\ \ \text{kg}\cdot\text{m}^2$$

The final angular velocity can then be determined following the solution to part (a) of Example 10.15: $\omega' = \dfrac{mvr}{I'} = \dfrac{(0.0500\ \text{kg})(30.0\ \text{m/s})(0.100\ \text{m})}{0.961\ \text{kg}\cdot\text{m}^2} = \underline{0.156\ \text{rad/s}}$

(b) The kinetic energy before the collision is the same as in Example 10.15: $\text{KE} = \underline{22.5\ \text{J}}$ The final kinetic energy is now:

$$\text{KE}' = \frac{1}{2}I'\omega_2'^{\,2} = \frac{1}{2}(0.961\ \text{kg}\cdot\text{m}^2)(0.156\ \text{rad/s})^2 = \underline{1.17\times10^{-2}\ \text{J}}$$

(c) The initial linear momentum is the same as in Example 10.15: $p = \underline{1.50\ \text{kg}\cdot\text{m/s}}$.

The final linear momentum is then

$$p' = mr\omega' + \frac{M}{2}R\omega' = \left[mr + \frac{M}{2}R\right]\omega', \text{so that:}$$

$$p' = \left[(0.0500\ \text{kg})(0.100\ \text{m}) + (1.00\ \text{kg})(1.20\ \text{m})\right]\cdot(0.156\ \text{rad/s}) = \underline{0.188\ \text{kg}\cdot\text{m/s}}$$

CHAPTER 11: FLUID STATICS

11.2 DENSITY

1. *Gold is sold by the troy ounce (31.103 g). What is the volume of 1 troy ounce of pure gold?*

Solution
From Table 11.1: $\rho_{\text{Au}} = 19.32\ \text{g/cm}^3$, so using the equation $\rho = \dfrac{m}{V}$, we have:

$$V = \frac{m}{\rho} = \frac{31.103\ \text{g}}{19.32\ \text{g/cm}^3} = \underline{1.610\ \text{cm}^3}$$

6. *(a) A rectangular gasoline tank can hold 50.0 kg of gasoline when full. What is the depth of the tank if it is 0.500-m wide by 0.900-m long? (b) Discuss whether this gas tank has a reasonable volume for a passenger car.*

Solution
(a) From Table 11.1: $\rho_{\text{gas}} = 0.680\times10^3\ \text{kg/m}^3$, so using the equation $\rho = \dfrac{m}{V} = \dfrac{m}{\rho l w}$,

the height is: $h = \dfrac{m}{\rho l w} = \dfrac{50.0\ \text{kg}}{(0.680\times10^3\ \text{kg/m}^3)(0.900\ \text{m})(0.500\ \text{m})} = \underline{0.163\ \text{m}}$

(b) The volume of this gasoline tank is 19.4 gallons, quite reasonably sized for a passenger car.

11.3 PRESSURE

12. *The pressure exerted by a phonograph needle on a record is surprisingly large. If the equivalent of 1.00 g is supported by a needle, the tip of which is a circle 0.200 mm in radius, what pressure is exerted on the record in* N/m^2*?*

Solution Using the equation $P = \frac{F}{A}$, we can solve for the pressure:

$$P = \frac{F}{A} = \frac{mg}{\pi r^2} = \frac{\left(1.00 \times 0^{-3}\ \text{kg}\right)\left(9.80\ \text{m/s}^2\right)}{\pi\left(2.00 \times 10^{-4}\ \text{m}\right)^2} = \underline{7.80 \times 10^4\ \text{Pa}}$$

This pressure is approximately 585 mm Hg.

11.4 VARIATION OF PRESSURE WITH DEPTH IN A FLUID

18. *The aqueous humor in a person's eye is exerting a force of 0.300 N on the* 1.10-cm^2 *area of the cornea. (a) What pressure is this in mm Hg? (b) Is this value within the normal range for pressures in the eye?*

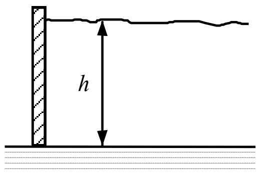
Solution (a) Using the equation $P = \frac{F}{A}$, we can solve for the pressure:

$$P = \frac{F}{A} = \frac{0.300\ \text{N}}{1.10\ \text{cm}^2} \times \left(\frac{100\ \text{cm}}{1\ \text{m}}\right)^2 = 2.73 \times 10^3\ \text{Pa} \times \frac{1\ \text{mm Hg}}{133.3\ \text{Pa}} = \underline{20.5\ \text{mm Hg}}$$

(b) From Table 11.5, we see that the range of pressures in the eye is 12-24 mm Hg, so the result in part (a) is within that range.

23. *Show that the total force on a rectangular dam due to the water behind it increases with the square of the water depth. In particular, show that this force is given by* $F = \rho g h^2 L/2$, *where* ρ *is the density of water,* h *is its depth at the dam, and* L *is the length of the dam. You may assume the face of the dam is vertical. (Hint: Calculate the average pressure exerted and multiply this by the area in contact with the water. See Figure 11.42.)*

Solution The average pressure on a dam is given by the equation $\overline{P} = \frac{h}{2}\rho g$, where $\frac{h}{2}$ is the average height of the water behind the dam. Then, the force on the dam is found using the equation $P = \frac{F}{A}$, so that $F = \bar{P}A = \left(\frac{h}{2}\rho g\right)(hL)$, or $F = \frac{\rho g h^2 L}{2}$. Thus, the average force on a rectangular dam increases with the square of the depth.



11.5 PASCAL'S PRINCIPLE

27. *A certain hydraulic system is designed to exert a force 100 times as large as the one put into it. (a) What must be the ratio of the area of the slave cylinder to the area of the master cylinder? (b) What must be the ratio of their diameters? (c) By what factor is the distance through which the output force moves reduced relative to the distance through which the input force moves? Assume no losses to friction.*

Solution (a) Using the equation $\frac{F_1}{A_1} = \frac{F_2}{A_2}$ we see that the ratio of the areas becomes:

$$\frac{A_S}{A_M} = \frac{F_S}{F_M} = \frac{100}{1} = \underline{100}$$

(b) We know that the area goes as $\pi r^2 = \frac{\pi d^2}{4}$, so the ratio of the areas gives:

$$\frac{A_S}{A_M} = \frac{\pi r_S^2}{\pi r_M^2} = \frac{\pi(d_S/2)^2}{\pi(d_M/2)^2} = \frac{d_S^2}{d_M^2} = 100, \text{so that } \frac{d_S}{d_M} = \sqrt{100} = \underline{10.0}$$

(c) Since the work input equals the work output, and work is proportional to force times distance, $F_i d_i = F_o d_o \Rightarrow \frac{d_o}{d_i} = \frac{F_i}{F_o} = \frac{1}{100}$.

This tells us that the distance through which the output force moves is reduced by a factor of 100, relative to the distance through which the input force moves.

28. *(a) Verify that work input equals work output for a hydraulic system assuming no losses to friction. Do this by showing that the distance the output force moves is reduced by the same factor that the output force is increased. Assume the volume of the fluid is constant. (b) What effect would friction within the fluid and between components in the system have on the output force? How would this depend on whether or not the fluid is moving?*

Solution (a) If the input cylinder is moved a distance d_i, it displaces a volume of fluid V, where the volume of fluid displaced must be the same for the input as the output: $V = d_\mathrm{i} A_\mathrm{i} = d_\mathrm{o} A_\mathrm{o} \Rightarrow d_\mathrm{o} = d_\mathrm{i}\left(\dfrac{A_\mathrm{i}}{A_\mathrm{o}}\right)$

Now, using the equation $\dfrac{F_1}{A_1} = \dfrac{F_2}{A_2}$, we can write the ratio of the areas in terms of the ratio of the forces: $\dfrac{F_1}{A_1} = \dfrac{F_2}{A_2} \Rightarrow F_\mathrm{o} = F_\mathrm{i}\left(\dfrac{A_\mathrm{o}}{A_\mathrm{i}}\right)$.

Finally, writing the output in terms of force and distance gives:

$$W_\mathrm{o} = F_\mathrm{o} d_\mathrm{o} = \left(\frac{F_\mathrm{i} A_\mathrm{o}}{A_\mathrm{i}}\right)\left(\frac{d_\mathrm{i} A_\mathrm{i}}{A_\mathrm{o}}\right) = F_\mathrm{i} d_\mathrm{i} = W_\mathrm{i}.$$

In other words, the work output equals the work input for a hydraulic system.

(b) If the system is not moving, the fraction would not play a role. With friction, we know there are losses, so that $W_\mathrm{o} = W_\mathrm{i} - W_\mathrm{f}$; therefore, the work output is less than the work input. In other words, with friction, you need to push harder on the input piston than was calculated. Note: the volume of fluid is still conserved.

11.7 ARCHIMEDES' PRINCIPLE

40. *Bird bones have air pockets in them to reduce their weight—this also gives them an average density significantly less than that of the bones of other animals. Suppose an ornithologist weighs a bird bone in air and in water and finds its mass is* $45.0\,\text{g}$ *and its apparent mass when submerged is* $3.60\,\text{g}$ *(the bone is watertight). (a) What mass of water is displaced? (b) What is the volume of the bone? (c) What is its average density?*

Solution (a) The apparent mass loss is equal to the mass of the fluid displaced, so the mass of the fluid displaced is just the difference the mass of the bone and its apparent mass: $m_{\text{displaced}} = 45.0\text{ g} - 3.60\text{ g} = \underline{41.4\text{ g}}$

(b) Using Archimedes' Principle, we know that that volume of water displaced equals the volume of the bone; we see that $V_{\text{b}} = V_{\text{w}} = \dfrac{m_{\text{w}}}{\rho_{\text{w}}} = \dfrac{41.4\text{ g}}{1.00\text{ g/cm}^3} = \underline{41.4\text{ cm}^3}$

(c) Using the following equation, we can calculate the average density of the bone:

$$\bar{\rho}_{\text{o}} = \frac{m_{\text{b}}}{V_{\text{b}}} = \frac{45.0\text{ g}}{41.4\text{ cm}^3} = \underline{1.09\text{ g/cm}^3}$$

This is clearly not the density of the bone everywhere. The air pockets will have a density of approximately $1.29\times10^{-3}\text{ g/cm}^3$, while the bone will be substantially denser.

46. *(a) What is the density of a woman who floats in freshwater with* 4.00% *of her volume above the surface? This could be measured by placing her in a tank with marks on the side to measure how much water she displaces when floating and when held under water (briefly). (b) What percent of her volume is above the surface when she floats in seawater?*

Solution (a) From the equation $\text{fraction submerged} = \dfrac{\bar{\rho}_{\text{obj}}}{\rho_{\text{fl}}}$, we see that:

$$\bar{\rho}_{\text{person}} = \rho_{\text{fresh water}} \times (\text{fraction submerged}) = \left(1.00 \times 10^3\ \text{kg/m}^3\right)(0.960) = \underline{960\ \text{kg/m}^3}$$

(b) The density of seawater is greater than that of fresh water, so she should float more.

$$\text{fraction submerged} = \frac{\bar{\rho}_{\text{person}}}{\rho_{\text{sea water}}} = \frac{960\ \text{kg/m}^3}{1025\ \text{kg/m}^3} = 0.9366.$$

Therefore, the percent of her volume above water is
$$\%\,\text{above water} = (1.0000 - 0.9366) \times 100\% = \underline{6.34\%}$$

She does indeed float more in seawater.

50. *Scurrilous con artists have been known to represent gold-plated tungsten ingots as pure gold and sell them to the greedy at prices much below gold value but deservedly far above the cost of tungsten. With what accuracy must you be able to measure the mass of such an ingot in and out of water to tell that it is almost pure tungsten rather than pure gold?*

Solution To determine if the ingot is gold or tungsten, we need to calculate the percent difference between the two substances both out and in water. Then, the difference between these percent differences is the necessary accuracy that we must have in order to determine the substance we have. The percent difference is calculated by calculating the difference in a quantity and dividing that by the value for gold.

<u>*Out of water*</u>: Using the difference in density, the percent difference is then:

$$\%_{\text{out}} = \frac{\rho_{\text{g}} - \rho_{\text{t}}}{\rho_{\text{g}}} \times 100\% = \frac{19.32\ \text{g/cm}^3 - 19.30\ \text{g/cm}^3}{19.32\ \text{g/cm}^3} \times 100\% = \underline{0.1035\%\ \text{in air}}$$

<u>*In water*</u>: Assume a $1.000\ \text{cm}^3$ nugget. Then the apparent mass loss is equal to that of the water displaced, i.e., 1.000 g. So, we can calculate the percent difference in the

mass loss by using the difference in masses:

$$\%_{\text{in}} = \frac{m'_{\text{g}} - m'_{\text{t}}}{m'_{\text{g}}} \times 100\% = \frac{18.32\ \text{g/cm}^3 - 18.30\ \text{g/cm}^3}{18.32\ \text{g/cm}^3} \times 100\% = \underline{0.1092\%\ \text{in water}}$$

The difference between the required accuracies for the two methods is $0.1092\ \% - 0.1035\ \% = \underline{0.0057\ \%} = \underline{0.006\ \%}$, so we need 5 digits of accuracy to determine the difference between gold and tungsten.

11.8 COHESION AND ADHESION IN LIQUIDS: SURFACE TENSION AND CAPILLARY ACTION

59. *We stated in Example 11.12 that a xylem tube is of radius $2.50 \times 10^{-5}\ \text{m}$. Verify that such a tube raises sap less than a meter by finding h for it, making the same assumptions that sap's density is $1050\ \text{kg/m}^3$, its contact angle is zero, and its surface tension is the same as that of water at $20.0°\text{C}$.*

Solution Use the equation $h = \frac{2\gamma\cos\theta}{\rho g r}$ to find the height to which capillary action will move sap through the xylem tube:

$$h = \frac{2\gamma\cos\theta}{\rho g r} = \frac{2(0.0728\ \text{N/m})(\cos 0°)}{(1050\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(2.50 \times 10^{-5}\ \text{m})} = \underline{0.566\ \text{m}}$$

65. *When two soap bubbles touch, the larger is inflated by the smaller until they form a single bubble. (a) What is the gauge pressure inside a soap bubble with a 1.50-cm radius? (b) Inside a 4.00-cm-radius soap bubble? (c) Inside the single bubble they form if no air is lost when they touch?*

Solution (a) Use the equation $P = \frac{4\gamma}{r}$ to find the gauge pressure inside a spherical soap bubble of radius 1.50 cm: $P_1 = \frac{4\gamma}{r} = \frac{4(0.0370\text{ N/m})}{(1.50\times 10^{-2}\text{ m})} = \underline{9.87\text{ N/m}^2}$

(b) Use $P = \frac{4\gamma}{r}$ to find the gauge pressure inside a spherical soap bubble of radius 4.00 cm: $P_2 = \frac{4\gamma}{r} = \frac{4(0.0370\text{ N/m})}{(0.0400\text{ m})} = \underline{3.70\text{ N/m}^2}$

(c) If they form one bubble without losing any air, then the total volume remains constant: $V = V_1 + V_2 = \frac{4}{3}\pi r_1^3 + \frac{4}{3}\pi r_2^3 = \frac{4}{3}\pi R^3$

Solving for the single bubble radius gives:

$$R = \left[r_1^3 + r_2^3\right]^{1/3} = \left[(0.0150\text{ m})^3 + (0.0400\text{ m})^3\right]^{1/3} = 0.0406\text{ m}.$$

So we can calculate the gauge pressure for the single bubble using the equation

$$P = \frac{4\gamma}{r} = \frac{4(0.0370\text{ N/m})}{0.0406\text{ m}} = \underline{3.65\text{ N/m}^2}$$

11.9 PRESSURES IN THE BODY

71. *Heroes in movies hide beneath water and breathe through a hollow reed (villains never catch on to this trick). In practice, you cannot inhale in this manner if your lungs are more than 60.0 cm below the surface. What is the maximum negative gauge pressure you can create in your lungs on dry land, assuming you can achieve* -3.00 cm *water pressure with your lungs 60.0 cm below the surface?*

Solution The negative gauge pressure that can be achieved is the sum of the pressure due to the water and the pressure in the lungs:

$$P = -3.00\text{ cm H}_2\text{O} - \left(-60.0\text{ cm H}_2\text{O}\right) = \underline{-63.0\text{ cm H}_2\text{O}}$$

75. *Pressure in the spinal fluid is measured as shown in Figure 11.43. If the pressure in the spinal fluid is 10.0 mm Hg: (a) What is the reading of the water manometer in cm water? (b) What is the reading if the person sits up, placing the top of the fluid 60 cm above the tap? The fluid density is 1.05 g/mL.*

Solution (a) This part is a unit conversion problem:

$$P_0 = (10.0\ \text{mm Hg})\left(\frac{133\ \text{N/m}^2}{1.0\ \text{mm Hg}}\right)\left(\frac{1.0\ \text{cm H}_2\text{O}}{98.1\ \text{N/m}^2}\right) = \underline{13.6\ \text{m H}_2\text{O}}$$

(b) Solving this part in standard units, we know that

$$P = P_0 + \Delta P = P_0 + h\rho g, \text{ or}$$
$$P = 1330\ \text{N/m}^2 + (1.05\times 10^3\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(0.600\ \text{m}) = 7504\ \text{N/m}^2$$

Then converting to cm water: $P = (7504\ \text{N/m}^2)\left(\frac{1.0\ \text{cm H}_2\text{O}}{98.1\ \text{N/m}^2}\right) = \underline{76.5\ \text{cm H}_2\text{O}}$

82. *Calculate the pressure due to the ocean at the bottom of the Marianas Trench near the Philippines, given its depth is* $11.0\ \text{km}$ *and assuming the density of sea water is constant all the way down. (b) Calculate the percent decrease in volume of sea water due to such a pressure, assuming its bulk modulus is the same as water and is constant. (c) What would be the percent increase in its density? Is the assumption of constant density valid? Will the actual pressure be greater or smaller than that calculated under this assumption?*

Solution (a) Using the equation $P = h\rho g$, we can calculate the pressure at a depth of 11.0 km:

$$P = h\rho g = (11.0\times 10^3\ \text{m})(1025\ \text{kg/m}^3)(9.80\ \text{m/s}^2)$$
$$= 1.105\times 10^8\ \text{N/m}^2 \times \frac{1\ \text{atm}}{1.013\times 10^5\ \text{N/m}^2} = \underline{1.09\times 10^3\ \text{atm}}$$

(b) Using the following equation:

$$\frac{\Delta V}{V_0} = \frac{1}{B}\frac{F}{A} = \frac{P}{B} = \frac{1.105 \times 10^8\ \text{N/m}^2}{2.2 \times 10^9\ \text{N/m}^2} = 5.02 \times 10^{-2} = \underline{5.0\%\ \text{decrease in volume}}.$$

(c) Using the equation $P = \frac{m}{V}$, we can get an expression for percent change in density: $\frac{\Delta\rho}{\rho} = \frac{m/(V_0 - \Delta V)}{m/V_0} = \frac{V_0}{V_0 - \Delta V} = \frac{1}{1-(\Delta V/V_0)} = \frac{1}{1.00 - 5.02 \times 10^{-2}} = 1.053,$

so that the percent increase in density is $\underline{5.3\%}$. Therefore, the assumption of constant density is not strictly valid. The actual pressure would be greater, since the pressure is proportional to density.

CHAPTER 12: FLUID DYNAMICS AND ITS BIOLOGICAL AND MEDICAL APPLICATIONS

12.1 FLOW RATE AND ITS RELATION TO VELOCITY

1. *What is the average flow rate in* cm^3/s *of gasoline to the engine of a car traveling at 100 km/h if it averages 10.0 km/L?*

Solution We are given the speed of the car and a gas mileage, giving us a volume consumed per time, so the equation $Q = \frac{V}{t}$ is the formula we want to use to calculate the average flow rate:

$$Q = \frac{V}{t} = \frac{\text{speed}}{\text{gas mileage}} = \frac{100\ \text{km/h}}{10.0\ \text{km/L}} \times \frac{1000\,\text{cm}^3}{1\,\text{L}} \times \frac{1\,\text{H}}{3600\,\text{s}} = \underline{2.78\ \text{cm}^3/\text{s}}$$

14. *Prove that the speed of an incompressible fluid through a constriction, such as in a Venturi tube, increases by a factor equal to the square of the factor by which the diameter decreases. (The converse applies for flow out of a constriction into a larger-diameter region.)*

Solution If the fluid is incompressible, then the flow rate through both sides will be equal: $Q = A_1 v_1 = A_2 v_2$. Writing the areas in terms of the diameter of the tube gives:

$$\pi \frac{d_1^2}{4} v_1 = \pi \frac{d_2^2}{4} v_2 \Rightarrow v_2 = v_1\left(d_1^2 / d_2^2\right) = \underline{v_1\left(d_1 / d_2\right)^2}$$

Therefore, the velocity through section 2 equals the velocity through section 1 times the square of the ratio of the diameters of section 1 and section 2.

12.2 BERNOULLI'S EQUATION

21. *Every few years, winds in Boulder, Colorado, attain sustained speeds of 45.0 m/s (about 100 mi/h) when the jet stream descends during early spring. Approximately what is the force due to the Bernoulli effect on a roof having an area of* $220\ \text{m}^2$*? Typical air density in Boulder is* $1.14\ \text{kg/m}^3$*, and the corresponding atmospheric pressure is* $8.89\times10^4\ \text{N/m}^2$*. (Bernoulli's principle as stated in the text assumes laminar flow. Using the principle here produces only an approximate result, because there is significant turbulence.)*

Solution Ignoring turbulence, we can use Bernoulli's equation:

$P_1+\frac{1}{2}\rho v_1^2+\rho g h_1 = P_2+\frac{1}{2}\rho v_2^2+\rho g h_2$,where the heights are the same: $h_1 = h_2$ because we are concerned about above and below a thin roof. The velocity inside the house is zero, so $v_1 = 0.0\ \text{m/s}$, while the speed outside the house is $v_2 = 45.0\ \text{m/s}$. The difference in pressures, $P_1 - P_2$, can then be found: $P_1 - P_2 = \frac{1}{2}\rho v_2^2$. Now, we can relate the change in pressure to the force on the roof, using the Equation $F=(P_1-P_2)A$, because we know the area of the roof $\left(A = 200\ \text{m}^2\right)$:

$$F=(P_1-P_2)A=\frac{1}{2}\rho\left(v_2^2-v_1^2\right)A$$

and substituting in the values gives:

$$F=\frac{1}{2}\left(1.14\ \text{kg/m}^3\right)\left[(45\ \text{m/s})^2-(0.0\ \text{m/s})^2\right]\left(220\ \text{m}^2\right)=\underline{2.54\times10^5\ \text{N}}$$

This extremely large force is the reason you should leave windows open in your home when there are tornados or heavy windstorms in the area: otherwise your roof will pop off!

12.3 THE MOST GENERAL APPLICATIONS OF BERNOULLI'S EQUATION

27. *The left ventricle of a resting adult's heart pumps blood at a flow rate of $83.0\ \text{cm}^3/\text{s}$, increasing its pressure by 110 mm Hg, its speed from zero to 30.0 cm/s, and its height by 5.00 cm. (All numbers are averaged over the entire heartbeat.) Calculate the total power output of the left ventricle. Note that most of the power is used to increase blood pressure.*

Solution Using the equation for power in fluid flow, we can calculate the power output by the left ventricle during the heartbeat:

$$\text{power} = \left(P + \frac{1}{2}\rho v^2 + \rho g h\right)Q, \text{where}$$

$$P = 110\ \text{mm Hg} \times \frac{133\ \text{N/m}^2}{1.0\ \text{mm Hg}} = 1.463\times10^4\ \text{N/m}^2,$$

$$\frac{1}{2}\rho v^2 = \frac{1}{2}\left(1.05\times10^3\ \text{kg/m}^3\right)\left(0.300\ \text{m/s}\right)^2 = 47.25\ \text{N/m}^2\text{, and}$$

$$\rho g h = \left(1.05\times10^3\ \text{kg/m}^3\right)\left(9.80\ \text{m/s}^2\right)\left(0.0500\ \text{m}\right) = 514.5\ \text{N/m}^2\text{, giving:}$$

$$\text{power} = \left(1.463\times10^4\ \text{N/m}^2 + 47.25\ \text{N/m}^2 + 514.5\ \text{N/m}^2\right)\left(83.0\ \text{cm}^3/\text{s}\right)\frac{10^{-6}\ \text{m}^3}{\text{cm}^3}$$

$$= \underline{1.26\ \text{W}}$$

12.4 VISCOSITY AND LAMINAR FLOW; POISEUILLE'S LAW

35. *The arterioles (small arteries) leading to an organ, constrict in order to decrease flow to the organ. To shut down an organ, blood flow is reduced naturally to 1.00% of its original value. By what factor did the radii of the arterioles constrict? Penguins do this when they stand on ice to reduce the blood flow to their feet.*

Solution If the flow rate is reduced to 1.00% of its original value, then

$Q_2 = \dfrac{\Delta P \pi r_2^4}{8\eta L_2} = 0.0100 Q_1 = 0.0100 \dfrac{\Delta P \pi r_1^4}{8\eta L_1}$. Since the length of the arterioles is kept constant and the pressure difference is kept constant, we can get a relationship between the radii: $r_2^4 = 0.0100 r_1^4 \Rightarrow r_2 = \left(0.0100\right)^{1/4} r_1 = \underline{0.316 r_1}$

The radius is reduced to 31.6% of the original radius to reduce the flow rate to 1.00% of its original value.

43. *Example 12.8 dealt with the flow of saline solution in an IV system. (a) Verify that a pressure of* $1.62\times10^4\ \text{N/m}^2$ *is created at a depth of 1.61 m in a saline solution, assuming its density to be that of sea water. (b) Calculate the new flow rate if the height of the saline solution is decreased to 1.50 m. (c) At what height would the direction of flow be reversed? (This reversal can be a problem when patients stand up.)*

Solution (a) We can calculate the pressure using the equation $P_2=\rho hg$ where the height is 1.61 m and the density is that of seawater:

$$P_2=\rho hg=\left(1025\ \text{kg/m}^3\right)\left(1.61\ \text{m}\right)\left(9.80\ \text{m/s}^2\right)=\underline{1.62\times10^4\ \text{N/m}^2}$$

(b) If the pressure is decreased to 1.50 m, we can use the equation $Q=\dfrac{(P_2-P_1)\pi r^4}{8\eta l}$ to determine the new flow rate: $Q=\dfrac{(P_2-P_1)\pi r^4}{8\eta l}$. We use $l=0.0250\ \text{m},\ r=0.150\times10^{-3}\ \text{m},\eta=1.005\times10^{-3}\ \text{N}\cdot\text{s/m}^2$, and $P_1=1.066\times10^3\ \text{N/m}^2$.

Using the equation $P_2=\rho hg$, we can find the pressure due to a depth of 1.50 m:

$$P_2'=\left(1025\ \text{kg/m}^3\right)\left(1.50\ \text{m}\right)\left(9.80\ \text{m/s}^2\right)=1.507\times10^4\ \text{N/m}^2.$$

So substituting into the equation $Q=\dfrac{(P_2-P_1)\pi r^4}{8\eta l}$ gives:

$$Q=\frac{\left[(1.507\times10^4\ \text{N/m}^2-1.066\times10^3\ \text{N/m}^2)\pi\left(0.150\times10^{-3}\ \text{m}\right)^4\right]}{8\left(1.005\times10^{-3}\ \text{N}\cdot\text{s/m}^2\right)\left(0.0250\ \text{m}\right)}$$
$$=1.11\times10^{-7}\ \text{m}^3/\text{s}=\underline{0.111\ \text{cm}^3/\text{s}}$$

(c) The flow rate will be zero (and become negative) when the pressure in the IV is equal to (or less than) the pressure in the patient's vein:

$$P_\text{r}=\rho hg\Rightarrow h=\frac{P_\text{r}}{\rho g}=\frac{1.066\times10^3\ \text{N/m}^2}{\left(1025\ \text{kg/m}^3\right)\left(9.80\ \text{m/s}^2\right)}=0.106\ \text{m}=\underline{10.6\ \text{cm}}$$

12.5 THE ONSET OF TURBULENCE

51. *Verify that the flow of oil is laminar (barely) for an oil gusher that shoots crude oil 25.0 m into the air through a pipe with a 0.100-m diameter. The vertical pipe is 50 m long. Take the density of the oil to be* $900\ \text{kg/m}^3$ *and its viscosity to be* $1.00\ (\text{N/m}^2)\cdot\text{s}$ *(or* $1.00\ \text{Pa}\cdot\text{s}$*).*

Solution We will use the equation $N_\text{R} = \dfrac{2\rho v r}{\eta}$ to determine the Reynolds number, so we must determine the velocity of the oil. Since the oil rises to 25.0 m,

$$v^2 = v_0^2 - 2gy\text{, where } v = 0\ \text{m/s}, y = 25.0\ \text{m}\text{, so}$$
$$v_0 = \sqrt{2gy} = \sqrt{2(9.80\ \text{m/s}^2)(25.0\ \text{m})} = 22.136\ \text{m/s}$$

Now, we can use the equation $N_\text{R} = \dfrac{2\rho v r}{\eta}$:

$$N_\text{R} = \frac{2(900\ \text{kg/m}^3)(22.136\ \text{m/s})(0.0500\ \text{m})}{1.00\ (\text{N/m}^2)\cdot\text{s}} = \underline{1.99\times10^3} < 2000$$

Since $N_\text{R} = 2000$is the approximate upper value for laminar flow. So the flow of oil is laminar (barely).

59. *Gasoline is piped underground from refineries to major users. The flow rate is* $3.00\times10^{-2}\ \text{m}^3/\text{s}$ *(about 500 gal/min), the viscosity of gasoline is* $1.00\times10^{-3}\ (\text{N/m}^2)\cdot\text{s}$*, and its density is* $680\ \text{kg/m}^3$*. (a) What minimum diameter must the pipe have if the Reynolds number is to be less than 2000? (b) What pressure difference must be maintained along each kilometer of the pipe to maintain this flow rate?*

Solution (a) We will use the equation $N_\text{R} = \dfrac{2\rho v r}{\eta}$, where $N_\text{R} = \dfrac{2\rho v r}{\eta} \le 2000$, to find the minimum radius, which will give us the minimum diameter. First, we need to get an expression for the velocity, from the equation $v = \dfrac{Q}{A} = \dfrac{Q}{\pi r^2}$. Substituting gives:

$\frac{2\rho\left(Q/\pi r^2\right)r}{\eta} = \frac{2\rho Q}{\pi\eta r} \le 2000$ or $r \ge \frac{\rho Q}{\pi\eta(1000)}$, so that the minimum diameter is

$$d \ge \frac{\rho Q}{500\pi\eta} = \frac{(680\ \text{kg/m}^3)(3.00\times10^{-2}\ \text{m}^3/\text{s})}{500\pi\left(1.00\times10^{-3}\ \text{N}\cdot\text{s/m}^2\right)} = \underline{13.0\ \text{m}}$$

(b) Using the equation $Q = \frac{\Delta P \pi r^4}{8\eta l}$, we can determine the pressure difference from the flow rate:

$$\Delta P = \frac{8\eta l Q}{\pi r^4} = \frac{8(1.00\times10^{-3}\ \text{N}\cdot\text{s/m}^2)(1000\ \text{m})(3.00\times10^{-2}\ \text{m}^3/\text{s})}{\pi\left(12.99\ \text{m}\right)^4} = \underline{2.68\times10^{-6}\ \text{N/m}^2}$$

This pressure is equivalent to $2.65\times10^{-11}\ \text{atm}$, which is very small pressure!

12.7 MOLECULAR TRANSPORT PHENOMENA: DIFFUSION, OSMOSIS, AND RELATED PROCESSES

66. *Suppose hydrogen and oxygen are diffusing through air. A small amount of each is released simultaneously. How much time passes before the hydrogen is 1.00 s ahead of the oxygen? Such differences in arrival times are used as an analytical tool in gas chromatography.*

Solution From Table 12.2, we know $D_{H_2} = 6.4\times10^{-5}\ \text{m}^2/\text{s}$ and $D_{O_2} = 1.8\times10^{-5}\ \text{m}^2/\text{s}$. We want to use the equation $x_{\text{rms}} = \sqrt{2Dt}$, since that relates time to the distance traveled during diffusion. We have two equations: $x_{\text{rms},O_2} = \sqrt{2D_{O_2}t_{O_2}}$ and $x_{\text{rms},H_2} = \sqrt{2D_{H_2}t_{H_2}}$.

We want the distance traveled to be the same, so we can set the equations equal. The distance will be the same when the time difference between t_{H_2} and t_{O_2} is 1.00 s, so we can relate the two times: $t_{O_2} = t_{H_2} + 1.00\ \text{s}$.

Setting the two distance equations equal and squaring gives: $2D_{O_2}t_{O_2} = 2D_{H_2}t_{H_2}$ and substituting for oxygen time gives: $D_{O_2}\left(t_{H_2} + 1.00\ \text{s}\right) = D_{H_2}t_{H_2}$.

Solving for the hydrogen time gives:

$$t_{H_2} = \frac{D_{O_2}}{D_{H_2} - D_{O_2}} \times 1.00\ \text{s} = \frac{1.8\times10^{-5}\ \text{m}^2/\text{s}}{6.4\times10^{-5}\ \text{m}^2/\text{s} - 1.8\times10^{-5}\ \text{m}^2/\text{s}} \times 1.00\ \text{s} = \underline{0.391\ \text{s}}$$

The hydrogen will take 0.391 s to travel to the distance x, while the oxygen will take

1.391 s to travel the same distance. Therefore, the hydrogen will be 1.00 seconds ahead of the oxygen after 0.391 s.

CHAPTER 13: TEMPERATURE, KINETIC THEORY, AND THE GAS LAWS

13.1 TEMPERATURE

1. *What is the Fahrenheit temperature of a person with a* $39.0°\text{C}$ *fever?*

Solution We can convert from Celsius to Fahrenheit:

$$T_{°\text{F}} = \frac{9}{5}(T_{°\text{C}}) + 32.0°$$

$$T_{°\text{F}} = \frac{9}{5}(39.0°\text{C}) + 32.0°\text{C} = \underline{102°\text{F}}$$

So $39.0°\text{C}$ is equivalent to $102°\text{F}$.

7. *(a) Suppose a cold front blows into your locale and drops the temperature by 40.0 Fahrenheit degrees. How many degrees Celsius does the temperature decrease when there is a* $40.0°\text{F}$ *decrease in temperature? (b) Show that any change in temperature in Fahrenheit degrees is nine-fifths the change in Celsius degrees.*

Solution (a) We can use the equation $T_{°\text{C}} = \frac{5}{9}(T_{°\text{F}} - 32)$ to determine the change in temperature.

$$\Delta T_{°\text{C}} = T_{°\text{C2}} - T_{°\text{C1}} = \frac{5}{9}(T_{°\text{F2}} - 32) - \frac{5}{9}(T_{°\text{F1}} - 32)$$

$$= \frac{5}{9}(T_{°\text{F2}} - T_{°\text{F1}}) = \frac{5}{9}\Delta T_{°\text{F}} = \frac{5}{9}(40°) = \underline{22.2°\text{C}}$$

(b) We know that $\Delta T_{°\text{F}} = T_{°\text{F2}} - T_{°\text{F1}}$. We also know that $T_{°\text{F2}} = \frac{9}{5}T_{°\text{C2}} + 32$ and $T_{°\text{F1}} = \frac{9}{5}T_{°\text{C1}} + 32$. So, substituting, we have $\Delta T_{°\text{F}} = \left(\frac{9}{5}T_{°\text{C2}} + 32\right) - \left(\frac{9}{5}T_{°\text{C1}} + 32\right)$.

Partially solving and rearranging the equation, we have $\Delta T_{°F} = \frac{9}{5}\left(T_{°C2} - T_{°C1}\right)$.

Therefore, $\underline{\Delta T_{°F} = \frac{9}{5}\Delta T_{°C}}$.

13.2 THERMAL EXPANSION OF SOLIDS AND LIQUIDS

15. *Show that 60.0 L of gasoline originally at* $15.0°C$ *will expand to 61.1 L when it warms to* $35.0°C$, *as claimed in Example 13.4.*

Solution We can get an expression for the change in volume using the equation $\Delta V = \beta V_0 \Delta T$, so the final volume is $V = V_0 + \Delta V = V_0(1 + \beta\Delta T)$, where $\beta = 9.50 \times 10^{-4}/°C$ for gasoline (see Table 13.2), so that

$$V' = V_0 + \beta V \Delta T = 60.0\,L + (9.50 \times 10^{-4}/°C)(60.0\,L)(20.0°C) = \underline{61.1\,L}$$

As the temperature is increased, the volume also increases.

21. *Show that* $\beta \approx 3\alpha$, *by calculating the change in volume* ΔV *of a cube with sides of length* L.

Solution From the equation $\Delta L = \alpha L_0 \Delta T$ we know that length changes with temperature. We also know that the volume of a cube is related to its length by $V = L^3$. Using the equation $V = V_0 + \Delta V$ and substituting for the sides we get $V = (L_0 + \Delta L)^3$. Then we replace ΔL with $\Delta L = \alpha L_0 \Delta T$ to get $V = (L_0 + \alpha L_0 \Delta T)^3 = L_0^3(1 + \alpha\Delta T)^3$. Since $\alpha\Delta T$ is small, we can use the binomial expansion to get $V = L_0^3(1 + 3\alpha\Delta T) = L_0^3 + 3\alpha L_0^3 \Delta T$. Rewriting the length terms in terms of volume gives $V = V_0 + \Delta V = V_0 + 3\alpha V_0 \Delta T$. By comparing forms we get $\Delta V = \beta V_0 \Delta T = 3\alpha V_0 \Delta T$. Thus, $\underline{\underline{\beta = 3\alpha}}$.

13.3 THE IDEAL GAS LAW

27. *In the text, it was shown that* $N/V = 2.68\times10^{25}\ \text{m}^{-3}$ *for gas at STP. (a) Show that this quantity is equivalent to* $N/V = 2.68\times10^{19}\ \text{cm}^{-3}$, *as stated. (b) About how many atoms are there in one* μm^3 *(a cubic micrometer) at STP? (c) What does your answer to part (b) imply about the separation of atoms and molecules?*

Solution (a) This is a units conversion problem, so

$$\frac{N}{V}=\left(\frac{2.68\times10^{25}}{\text{m}^3}\right)\left(\frac{1\ \text{m}}{100\ \text{cm}}\right)^3=\underline{2.68\times10^{19}\ \text{cm}^{-3}}$$

(b) Again, we need to convert the units:

$$\frac{N}{V}=\left(\frac{2.68\times10^{25}}{\text{m}^3}\right)\left(\frac{1\text{m}}{1.00\times10^{6}\ \mu\text{m}}\right)^3=\underline{2.68\times10^{7}\ \mu\text{m}^{-3}}$$

(c) This says that atoms and molecules must be on the order of (if they were tightly packed) $V=\dfrac{N}{2.68\times10^{7}\ \mu\text{m}^{-3}}=\dfrac{1}{2.68\times10^{7}\ \mu\text{m}^{-3}}=\underline{3.73\times10^{-8}\ \mu\text{m}^3}$

Or the average length of an atom is less than approximately $(3.73\times10^{-8}\ \mu\text{m}^3)^{1/3}=3.34\times10^{-3}\ \mu\text{m}=\underline{3\ \text{nm}}$.

Since atoms are widely spaced, the average length is probably more on the order of 0.3 nm.

33. *A bicycle tire has a pressure of* $7.00\times10^5\ \text{N/m}^2$ *at a temperature of* $18.0°\text{C}$ *and contains 2.00 L of gas. What will its pressure be if you let out an amount of air that has a volume of* $100\ \text{cm}^3$ *at atmospheric pressure? Assume tire temperature and volume remain constant.*

Solution First, we need to convert the temperature and volume:

$$T(\text{K})=T(°\text{C})+273.15=18.0+273.15=291.2\ \text{K, and}$$
$$V=2.00\ \text{L}=2.00\times10^{-3}\ \text{m}^3.$$

Next, use the ideal gas law to determine the initial number of molecules in the tire:

$$P_1V=N_1kT\Rightarrow N_1=\frac{P_1V}{kT}=\frac{(7.00\times10^5\ \text{N/m}^2)(2.00\times10^{-3}\ \text{m}^3)}{(1.38\times10^{-23}\ \text{J/K})(291.15\ \text{K})}=3.484\times10^{23}$$

Then, we need to determine how many molecules were removed from the tire:

$$PV = \Delta NkT \Rightarrow \Delta N = \frac{PV}{kT} = \frac{(1.013\times 10^5\ \text{N/m}^2)\left(100\ \text{cm}^3 \times \dfrac{10^{-6}\ \text{m}^3}{\text{cm}^3}\right)}{(1.38\times 10^{-23}\ \text{J/K})(291.15\text{K})} = 2.521\times 10^{21}$$

We can now determine how many molecules remain after the gas is released:

$$N_2 = N_1 - \Delta N = 3.484\times 10^{23} - 2.521\times 10^{21} = 3.459\times 10^{23}$$

Finally, the final pressure is:

$$P_2 = \frac{N_2 kT}{V} = \frac{(3.459\times 10^{23})(1.38\times 10^{-23}\ \text{J/K})(291.15\ \text{K})}{2.00\times 10^{-3}\ \text{m}^3}$$
$$= 6.95\times 10^5\ \text{N/m}^2 = \underline{6.95\times 10^5\ \text{Pa}}$$

38. *(a) In the deep space between galaxies, the density of atoms is as low as* $10^6\ \text{atoms/m}^3$*, and the temperature is a frigid 2.7 K. What is the pressure? (b) What volume (in* m^3*) is occupied by 1 mol of gas? (c) If this volume is a cube, what is the length of its sides in kilometers?*

Solution (a) Use the ideal gas law, where

$$PV = NkT$$
$$P = \frac{N}{V}kT = \frac{10^6}{1\ \text{m}^3}(1.38\times 10^{-23}\ \text{J/K})(2.7\ \text{K})$$
$$= 3.73\times 10^{-17}\ \text{N/m}^2 = 3.7\times 10^{-17}\ \text{N/m}^2 = \underline{3.7\times 10^{-17}\ \text{Pa}}$$

(b) Now, using the pressure found in part (a), let n = 1.00 mol. Use the ideal gas law:

$$PV = nRT$$
$$V = \frac{nRT}{P} = \frac{(1.00\ \text{mol})(8.31\ \text{J/mol}\cdot\text{K})(2.7\ \text{K})}{3.73\times 10^{-17}\ \text{N/m}^2} = 6.02\times 10^{17}\ \text{m}^3 = \underline{6.0\times 10^{17}\ \text{m}^3}$$

(c) Since the volume of a cube is its length cubed:

$$L = V^{1/3} = (6.02\times 10^{17}\ \text{m}^3)^{1/3} = 8.45\times 10^5\ \text{m} = \underline{8.4\times 10^2\ \text{km}}$$

13.4 KINETIC THEORY: ATOMIC AND MOLECULAR EXPLANATION OF PRESSURE AND TEMPERATURE

44. *Nuclear fusion, the energy source of the Sun, hydrogen bombs, and fusion reactors, occurs much more readily when the average kinetic energy of the atoms is high—that is, at high temperatures. Suppose you want the atoms in your fusion experiment to have average kinetic energies of* $6.40\times10^{-14}\ \text{J}$*. What temperature is needed?*

Solution Use the equation $\overline{\text{KE}}=\frac{3}{2}kT$ to find the temperature:

$$\overline{\text{KE}}=\frac{3}{2}kT \Rightarrow T=\frac{2\overline{\text{KE}}}{3k}=\frac{2\left(6.40\times10^{-14}\ \text{J}\right)}{3\left(1.38\times10^{-23}\ \text{J/K}\right)}=\underline{3.09\times10^{9}\ \text{K}}$$

13.6 HUMIDITY, EVAPORATION, AND BOILING

50. *(a) What is the vapor pressure of water at* $20.0°\text{C}$*? (b) What percentage of atmospheric pressure does this correspond to? (c) What percent of* $20.0°\text{C}$ *air is water vapor if it has 100% relative humidity? (The density of dry air at* $20.0°\text{C}$ *is* $1.20\ \text{kg/m}^3$*.)*

Solution (a) Vapor Pressure for $H_2O(20°\text{C})=2.33\times10^{3}\ \text{N/m}^2=\underline{2.33\times10^{3}\ \text{Pa}}$

(b) Divide the vapor pressure by atmospheric pressure:

$$\frac{2.33\times10^{3}\ \text{N/m}^2}{1.01\times10^{5}\ \text{N/m}^2}\times100\%=\underline{2.30\%}$$

(c) The density of water in this air is equal to the saturation vapor density of water at this temperature, taken from Table 13.5. Dividing by the density of dry air, we can get the percentage of water in the air: $\frac{1.72\times10^{-2}\ \text{kg/m}^3}{1.20\ \text{kg/m}^3}\times100\%=\underline{1.43\%}$

56. *What is the density of water vapor in* g/m^3 *on a hot dry day in the desert when the temperature is* $40.0°\text{C}$ *and the relative humidity is 6.00%?*

Solution

$$\text{percent relative humidity} = \frac{\text{vapor density}}{\text{saturation vapor density}} \times 100\%$$

$$\text{vapor density} = \frac{(\text{percent relative humidity})(\text{saturation vapor density})}{100\%}$$

$$= \frac{(6.00\%)(51.1\ \text{g/m}^3)}{100\%} = \underline{3.07\ \text{g/m}^3}$$

62. *Atmospheric pressure atop Mt. Everest is* $3.30 \times 10^4\ \text{N/m}^2$. *(a) What is the partial pressure of oxygen there if it is 20.9% of the air? (b) What percent oxygen should a mountain climber breathe so that its partial pressure is the same as at sea level, where atmospheric pressure is* $1.01 \times 10^5\ \text{N/m}^2$ *? (c) One of the most severe problems for those climbing very high mountains is the extreme drying of breathing passages. Why does this drying occur?*

Solution (a) The partial pressure is the pressure a gas would create if it alone occupied the total volume, or the partial pressure is the percent the gas occupies times the total pressure:

$$\text{partial pressure}(O_2) = (\%O_2)(\text{atmospheric pressure})$$
$$= (0.209)(3.30 \times 10^4\ \text{N/m}^2) = \underline{6.90 \times 10^3\ \text{Pa}}$$

(b) First calculate the partial pressure at sea level:

$$\text{partial pressure}(\text{at sea level}) = (\%O_2)(\text{atmospheric pressure})$$
$$= (0.209)(1.013 \times 10^5\ \text{N/m}^2) = \underline{2.117 \times 10^4\ \text{Pa}}$$

Set this equal to the percent oxygen times the pressure at the top of Mt. Everest:

$$\text{partial pressure}(\text{at sea level}) = \left(\frac{\%O_2}{100\%}\right)(3.30 \times 10^4\ \text{N/m}^2) = 2.117 \times 10^4\ \text{Pa}$$

Thus, $\%O_2 = \dfrac{2.117 \times 10^4\ \text{N/m}^2}{3.30 \times 10^4\ \text{N/m}^2} \times 100\% = \underline{64.2\%}$

The mountain climber should breathe air containing 64.2% oxygen at the top of Mt. Everest to maintain the same partial pressure as at sea level. Clearly, the air does not contain that much oxygen. This is why you feel lightheaded at high altitudes: You are partially oxygen deprived.

(c) This drying process occurs because the partial pressure of water vapor at high altitudes is decreased substantially. The climbers breathe very dry air, which leads to a lot of moisture being lost due to evaporation. The breathing passages are therefore not getting the moisture they require from the air being breathed.

68. ***Integrated Concepts*** *If you want to cook in water at* $150°\text{C}$*, you need a pressure cooker that can withstand the necessary pressure. (a) What pressure is required for the boiling point of water to be this high? (b) If the lid of the pressure cooker is a disk 25.0 cm in diameter, what force must it be able to withstand at this pressure?*

Solution (a) From Table 13.5, we can get the vapor pressure of water at $150°\text{C}$: Vapor pressure $= \underline{4.76\times10^5\ \text{N/m}^2}$

(b) Using the equation $P = \dfrac{F}{A}$, we can calculate the force exerted on the pressure cooker lid. Here, we need to use Newton's laws to balance forces. Assuming that we are cooking at sea level, the forces on the lid will stem from the internal pressure, found in part (a), the ambient atmospheric pressure, and the forces holding the lid shut. Thus we have a "balance of pressures":

$$P + 1\,\text{atm} - (4.76\times10^5\ \text{Pa}) = 0 \Rightarrow P = 3.75\times10^5\ \text{Pa}$$

$$\text{net } F = PA = (3.75\times10^5\ \text{Pa})(\pi(0.125\,\text{m})^2) = \underline{1.84\times10^4\ \text{N}}$$

CHAPTER 14: HEAT AND HEAT TRANSFER METHODS

14.2 TEMPERATURE CHANGE AND HEAT CAPACITY

1. *On a hot day, the temperature of an 80,000-L swimming pool increases by* $1.50°\text{C}$*. What is the net heat transfer during this heating? Ignore any complications, such as loss of water by evaporation.*

Solution The heat input is given by $Q = mc\Delta T$, where the specific heat of water is $c = 4186\ \text{J/kg}\cdot°\text{C}$. The mass is given by

$$m = \rho V = (1.00\times10^3\ \text{kg/m}^3)(80{,}000\ \text{L})\times\frac{1\ \text{m}^3}{1000\ \text{L}} = 8.00\times10^4\ \text{kg},$$

and the temperature change is $\Delta T = 1.50°\text{C}$. Therefore,

$Q = mc\Delta T = (8.00\times10^4\ \text{kg})(4186\ \text{J/kg}\cdot°\text{C})(1.50°\text{C}) = \underline{5.02\times10^8\ \text{J}}$

9. *Following vigorous exercise, the body temperature of an 80.0-kg person is* $40.0°\text{C}$*. At what rate in watts must the person transfer thermal energy to reduce the body temperature to* $37.0°\text{C}$ *in 30.0 min, assuming the body continues to produce energy at the rate of 150 W?* $(1\ \text{watt} = 1\ \text{joule/second or } 1\ \text{W} = 1\ \text{J/s})$

Solution First, calculate how much heat must be dissipated:

$$Q = mc_{\text{human body}}\Delta T = (80.0\ \text{kg})(3500\ \text{J/kg}\cdot°\text{C})(40°\text{C} - 37°\text{C}) = 8.40\times10^5\ \text{J}$$

Then, since power is heat divided by time, we can get the power required to produce the calculated amount of heat in 30.0 minutes:

$$P_{\text{cooling}} = \frac{Q}{t} = \frac{8.40 \times 10^5 \text{ J}}{(30 \text{ min})(60 \text{ s/1 min})} = 4.67 \times 10^2 \text{ W}.$$

Now, since the body continues to produce heat at a rate of 150 W, we need to add that to the required cooling power:

$$P_{\text{required}} = P_{\text{cooling}} + P_{\text{body}} = 467 \text{ W} + 150 \text{ W} = \underline{617 \text{ W}}.$$

14.3 PHASE CHANGE AND LATENT HEAT

12. *A bag containing $0°\text{C}$ ice is much more effective in absorbing energy than one containing the same amount of $0°\text{C}$ water. (a) How much heat transfer is necessary to raise the temperature of 0.800 kg of water from $0°\text{C}$ to $30.0°\text{C}$? (b) How much heat transfer is required to first melt 0.800 kg of $0°\text{C}$ ice and then raise its temperature? (c) Explain how your answer supports the contention that the ice is more effective.*

Solution (a) Use $Q = mc\Delta T$, since there is no phase change involved in heating the water:

$$Q = mc\Delta T = (0.800 \text{ kg})(4186 \text{ J/kg} \cdot °\text{C})(30.0°\text{C}) = \underline{1.00 \times 10^5 \text{ J}}$$

(b) To determine the heat required, we must melt the ice, using $Q = mL_\text{f}$, and then add the heat required to raise the temperature of melted ice using $Q = mc\Delta T$, so that $Q = mL_\text{f} + mc\Delta T = (0.800 \text{ kg})(334 \times 10^3 \text{ J/kg}) + 1.005 \times 10^5 \text{ J} = \underline{3.68 \times 10^5 \text{ J}}$

(c) The ice is much more effective in absorbing heat because it first must be melted, which requires a lot of energy, then it gains the heat that the water also would. The first $2.67 \times 10^5 \text{ J}$ of heat is used to melt the ice, then it absorbs the $1.00 \times 10^5 \text{ J}$ of heat that the water absorbs.

19. *How many grams of coffee must evaporate from 350 g of coffee in a 100-g glass cup to cool the coffee from* $95.0°\text{C}$ *to* $45.0°\text{C}$*? You may assume the coffee has the same thermal properties as water and that the average heat of vaporization is 2340 kJ/kg (560 cal/g). (You may neglect the change in mass of the coffee as it cools, which will give you an answer that is slightly larger than correct.)*

Solution The heat gained in evaporating the coffee equals the heat leaving the coffee and glass to lower its temperature, so that $ML_\text{v} = m_\text{c}c_\text{c}\Delta T + m_\text{g}c_\text{g}\Delta T,$ where M is the mass of coffee that evaporates. Solving for the evaporated coffee gives:

$$M = \frac{\Delta T(m_\text{c}c_\text{c} + m_\text{g}c_\text{g})}{L_\text{v}}$$
$$= \frac{(95.0°\text{C} - 45.0°\text{C})}{560\ \text{cal/g}} \cdot \left[(350\ \text{g})(1.00\ \text{cal/g}\cdot°\text{C}) + (100\ \text{g})(0.20\ \text{cal/g}\cdot°\text{C})\right] = \underline{33.0\ \text{g}}$$

Notice that we did the problem in calories and grams, since the latent heat was given in those units, and the result we wanted was in grams. We could have done the problem in standard units, and then converted back to grams to get the same answer.

25. *If you pour 0.0100 kg of* $20.0°\text{C}$ *water onto a 1.20-kg block of ice (which is initially at* $-15.0°\text{C}$*), what is the final temperature? You may assume that the water cools so rapidly that effects of the surroundings are negligible.*

Solution The heat gained by the ice equals the heat lost by the water. Since we do not know the final state of the water/ice combination, we first need to compare the heat needed to raise the ice to $0°\text{C}$ and the heat available from the water. First, we need to calculate how much heat would be required to raise the temperature of the ice to $0°\text{C}$: $Q_\text{ice} = mc\Delta T = (1.20\ \text{kg})(2090\ \text{J/kg}\cdot°\text{C})(15°\text{C}) = 3.762\times10^4\ \text{J}$

Now, we need to calculate how much heat is given off to lower the water to $0°\text{C}$:
$Q_1 = mc\Delta T_1 = (0.0100\,\text{kg})(4186\,\text{J/kg}\cdot°\text{C})(20.0°\text{C}) = 837.2\,\text{J}$

Since this is less than the heat required to heat the ice, we need to calculate how much heat is given off to convert the water to ice:

$Q_2 = mL_f = (0.0100\ \text{kg})(334\times10^3\ \text{J/kg}) = 3.340\times10^3\ \text{J}$

Thus, the total amount of heat given off to turn the water to ice at $0°\text{C}$:
$Q_{\text{water}} = 4.177\times10^3\ \text{J}$.

Since $Q_{\text{ice}} > Q_{\text{water}}$, we have determined that the final state of the water/ice is ice at some temperature below $0°\text{C}$. Now, we need to calculate the final temperature. We set the heat lost from the water equal to the heat gained by the ice, where we now know that the final state is ice at $T_f < 0°\text{C}$:

$Q_{\text{lost by water}} = Q_{\text{gained by ice,}}$ or $m_{\text{water}}c_{\text{water}}\Delta T_{20\to0} + m_{\text{water}}L_f + m_{\text{water}}c_{\text{ice}}\Delta T_{0\to?} = m_{\text{ice}}c_{\text{ice}}\Delta T_{-15\to?}$

Substituting for the change in temperatures (being careful that ΔT is always positive) and simplifying gives $m_{\text{water}}[c_{\text{water}}(20°\text{C}) + L_f + (c_{\text{ice}})(0 - T_f)] = m_{\text{ice}}c_{\text{ice}}[T_f - (-15°\text{C})]$.

Solving for the final temperature gives $T_f = \dfrac{m_{\text{water}}[c_{\text{water}}(20°\text{C}) + L_f] - m_{\text{ice}}c_{\text{ice}}(15°\text{C})}{(m_{\text{water}} + m_{\text{ice}})c_{\text{ice}}}$

and so finally,

$$\begin{aligned} T_f &= \frac{(0.0100\ \text{kg})[(4186\ \text{J/kg}\cdot°\text{C})(20°\text{C}) + 334\times10^3\ \text{J/kg}]}{(0.0100\ \text{kg} + 1.20\ \text{kg})(2090\ \text{J/kg}\cdot°\text{C})} \\ &\quad - \frac{(1.20\ \text{kg})(2090\ \text{J/kg}\cdot°\text{C})(15°\text{C})}{(0.0100\ \text{kg} + 1.20\ \text{kg})(2090\ \text{J/kg}\cdot°\text{C})} \\ &= \underline{-13.2°\text{C}} \end{aligned}$$

14.5 CONDUCTION

34. *A man consumes 3000 kcal of food in one day, converting most of it to maintain body temperature. If he loses half this energy by evaporating water (through breathing and sweating), how many kilograms of water evaporate?*

Solution Use $Q = mL_v$, where $Q = \frac{1}{2}3000\text{ kcal}$ and $L_{v(37°C)} = 580\text{ kcal/kg}$,

$$Q = mL_{v(37°C)} \Rightarrow m = \frac{Q}{L_{v(37°C)}} = \frac{1500\text{ kcal}}{580\text{ kcal/kg}} = \underline{2.59\text{ kg}}$$

38. *Compare the rate of heat conduction through a 13.0-cm-thick wall that has an area of* 10.0 m^2 *and a thermal conductivity twice that of glass wool with the rate of heat conduction through a window that is 0.750 cm thick and that has an area of* 2.00 m^2*, assuming the same temperature difference across each.*

Solution Use the rate of heat transfer by conduction, $\frac{Q}{t} = \frac{kA(T_2 - T_1)}{d}$, and take the ratio of the wall to the window. The temperature difference for the wall and the window will be the same:

$$\frac{(Q/t)_{\text{wall}}}{(Q/t)_{\text{window}}} = \frac{k_{\text{wall}}A_{\text{wall}}d_{\text{window}}}{k_{\text{window}}A_{\text{window}}d_{\text{wall}}}$$
$$= \frac{(2\times 0.042\text{ J/s}\cdot\text{m}\cdot°\text{C})(10.0\text{ m}^2)(0.750\times 10^{-2}\text{ m})}{(0.84\text{ J/s}\cdot\text{m}\cdot°\text{C})(2.00\text{ m}^2)(13.0\times 10^{-2}\text{ m})}$$
$$= 0.0288\text{ wall : window, or }\underline{35:1\text{ window : wall}}$$

So windows conduct more heat than walls. This should seem reasonable, since in the winter the windows feel colder than the walls.

14.6 CONVECTION

50. *One winter day, the climate control system of a large university classroom building malfunctions. As a result,* 500 m^3 *of excess cold air is brought in each minute. At what rate in kilowatts must heat transfer occur to warm this air by* $10.0°\text{C}$ *(that is, to bring the air to room temperature)?*

Solution Use $Q = mc\Delta T$ in combination with the equations $P = \frac{Q}{t}$ and $m = \rho V$ to get:

$$P = \frac{Q}{t} = \frac{mc\Delta T}{t} = \frac{\rho Vc\Delta T}{t} = \frac{(1.29\ \text{kg/m}^3)(500\ \text{m}^3)(721\ \text{J/kg}\cdot°\text{C})(10.0°\text{C})}{60.0\ \text{s}}$$
$$= 7.75\times10^4\ \text{W} = \underline{77.5\ \text{kW}}.$$

14.7 RADIATION

58. *(a) Calculate the rate of heat transfer by radiation from a car radiator at* $110°\text{C}$ *into a* $50.0°\text{C}$ *environment, if the radiator has an emissivity of 0.750 and a* 1.20 - m^2 *surface area. (b) Is this a significant fraction of the heat transfer by an automobile engine? To answer this, assume a horsepower of 200 hp (1.5 kW) and the efficiency of automobile engines as* 25%.

Solution (a) Using the Stefan-Boltzmann law of radiation, making sure to convert the temperatures into units of Kelvin, the rate of heat transfer is:

$$\frac{Q}{t} = \sigma eA(T_2^4 - T_1^4)$$
$$= (5.67\times10^{-8}\ \text{J/s}\cdot\text{m}^2\cdot\text{K}^4)(0.750)(1.20\ \text{m}^2)\left[(323\ \text{K})^4 - (383\ \text{K})^4\right] = \underline{-543\ \text{W}}$$

(b) Assuming an automobile engine is 200 horsepower and the efficiency of a gasoline engine is 25%, the engine consumes $\frac{200\ \text{horsepower}}{25\%} = 800$ horsepower in order to generate the 200 horsepower. Therefore, 800 horsepower is lost due to heating. Since 1 hp = 746 W, the radiator transfers $543\ \text{W}\times\frac{1\ \text{hp}}{746\ \text{W}} = 0.728\ \text{hp}$ from radiation, which is not a significant fraction because the heat is primarily transferred from the radiator by other means.

65. *(a) A shirtless rider under a circus tent feels the heat radiating from the sunlit portion of the tent. Calculate the temperature of the tent canvas based on the following information: The shirtless rider's skin temperature is* $34.0°\text{C}$ *and has an emissivity of 0.970. The exposed area of skin is* $0.400\ \text{m}^2$. *He receives radiation at the rate of 20.0 W—half what you would calculate if the entire region behind him was hot. The rest of the surroundings are at* $34.0°\text{C}$. *(b) Discuss how this situation would change if the sunlit side of the tent was nearly pure white and if the rider was covered by a white tunic.*

Solution (a) Use the Stefan-Boltzmann law of radiation:

$$\frac{Q}{t} = \sigma e\left(\frac{A}{2}\right)(T_2^4 - T_1^4),$$

$$T_2 = \left[\frac{Q/t}{\sigma e(A/2)} + T_1^4\right]^{1/4} = \left[\frac{2(Q/t)}{\sigma e A} + T_1^4\right]^{1/4}$$

$$= \left[\frac{2(20.0\ \text{W})}{(5.67\times10^{-8}\ \text{J/s}\cdot\text{m}^2\cdot\text{K}^4)(0.970)(0.400\ \text{m}^2)} + (307\ \text{K})^4\right]^{1/4}$$

$$= 321.63\ \text{K} = \underline{48.5°\text{C}}$$

(b) A pure white object reflects more of the radiant energy that hits it, so the white tent would prevent more of the sunlight from heating up the inside of the tent, and the white tunic would prevent that radiant energy inside the tent from heating the rider. Therefore, with a white tent, the temperature would be lower than $48.5°\text{C}$, and the rate of radiant heat transferred to the rider would be less than 20.0 W.

70. ***Integrated Concepts*** *(a) Suppose you start a workout on a Stairmaster, producing power at the same rate as climbing 116 stairs per minute. Assuming your mass is 76.0 kg and your efficiency is* 20.0%, *how long will it take for your body temperature to rise* 1.00°C *if all other forms of heat transfer in and out of your body are balanced? (b) Is this consistent with your experience in getting warm while exercising?*

Solution (a) You produce power at a rate of 685 W, and since you are 20% efficient, you must have generated: $P_{\text{generated}} = \dfrac{P_{\text{produced}}}{\text{efficiency}} = \dfrac{685\ \text{W}}{0.20} = 3425\ \text{W}$.

If only 685 W of power was useful, the power available to heat the body is $P_{\text{wasted}} = 3425\ \text{W} - 685\ \text{W} = 2.740 \times 10^3\ \text{W}$.

Now, $P_{\text{wasted}} = \dfrac{Q}{t} = \dfrac{mc\Delta T}{t}$, so that

$$t = \frac{mc\Delta T}{P_{\text{wasted}}} = \frac{(76.0\ \text{kg})(3500\ \text{J/kg}\cdot{}^\circ\text{C})(1.00^\circ\text{C})}{2.74\times 10^3\ \text{W}} = \underline{97.1\ \text{s}}$$

(b) This says that it takes about a minute and a half to generate enough heat to raise the temperature of your body by 1.00°C, which seems quite reasonable. Generally, within five minutes of working out on a Stairmaster, you definitely feel warm and probably are sweating to keep your body from overheating.

76. ***Unreasonable Results*** *(a) What is the temperature increase of an 80.0 kg person who consumes 2500 kcal of food in one day with* 95.0% *of the energy transferred as heat to the body? (b) What is unreasonable about this result? (c) Which premise or assumption is responsible?*

Solution (a) $Q = mc\Delta T, \text{ so that } \Delta T = \dfrac{Q}{mc} = \dfrac{(0.950)(2500\ \text{kcal})}{(80.0\ \text{kg})(0.83\ \text{kcal/kg}\cdot{}^\circ\text{C})} = \underline{36^\circ\text{C}}.$

This says that the temperature of the person is $37^\circ\text{C} + 36^\circ\text{C} = 73^\circ\text{C}$!

(b) Any temperature increase greater than about 3°C would be unreasonably large. In this case the final temperature of the person would rise to $73^\circ\text{C}\,(163^\circ\text{F})$.

(c) The assumption that the person retains 95% of the energy as body heat is unreasonable. Most of the food consumed on a day is converted to body heat, losing energy by sweating and breathing, etc.

CHAPTER 15: THERMODYNAMICS

15.1 THE FIRST LAW OF THERMODYNAMICS

1. *What is the change in internal energy of a car if you put 12.0 gal of gasoline into its tank? The energy content of gasoline is* $1.3\times10^8\ \text{J/gal}$*. All other factors, such as the car's temperature, are constant.*

Solution Using the energy content of a gallon of gasoline $1.3\times10^8\ \text{J/gal}$, the energy stored in 12.0 gallons of gasoline is: $E_{\text{gas}} = (1.3\times10^8\ \text{J/gal})(12.0\ \text{gal}) = 1.6\times10^9\ \text{J}$. Therefore, the internal energy of the car increases by this energy, so that $\Delta U = \underline{1.6\times10^9\ \text{J}}$.

7. *(a) What is the average metabolic rate in watts of a man who metabolizes 10,500 kJ of food energy in one day? (b) What is the maximum amount of work in joules he can do without breaking down fat, assuming a maximum efficiency of 20.0%? (c) Compare his work output with the daily output of a 187-W (0.250-horsepower) motor.*

Solution (a) The metabolic rate is the power, so that:

$$P = \frac{Q}{t} = \frac{10500\ \text{kJ}}{1\ \text{day}} \times \frac{1000\ \text{J}}{1\ \text{kJ}} \times \frac{1\ \text{day}}{8.64\times10^4\ \text{s}} = \underline{122\ \text{W}}$$

(b) Efficiency is defined to be the ratio of what we get to what we spend, or $Eff = \frac{W}{E_{\text{in}}}$, so we can determine the work done, knowing the efficiency:

$$W = Eff \cdot E_{\text{in}} = 0.200(10500\ \text{kJ}) \times \frac{1000\ \text{J}}{1\ \text{kJ}} = \underline{2.10\times10^6\ \text{J}}$$

(c) To compare with a 0.250 hp motor, we need to know how much work the motor does in one day: $W = Pt = (0.250\text{ hp})(1\text{ day})\times\dfrac{746\text{ W}}{1\text{ hp}}\times\dfrac{8.64\times10^4\text{ s}}{1\text{ day}} = 1.61\times10^7\text{ J}$.

So, the man's work output is: $\dfrac{W_{\text{motor}}}{W_{\text{man}}} = \dfrac{1.61\times10^7\text{ J}}{2.10\times10^6\text{ J}} = 7.67 \Rightarrow$ 7.67 times less than the motor. Thus the motor produces 7.67 times the work done by the man.

15.2 THE FIRST LAW OF THERMODYNAMICS AND SOME SIMPLE PROCESSES

11. *A helium-filled toy balloon has a gauge pressure of 0.200 atm and a volume of 10.0 L. How much greater is the internal energy of the helium in the balloon than it would be at zero gauge pressure?*

Solution First, we must assume that the volume remains constant, so that $V_1 = V_2$, where state 1 is that at $P_1 = 0.200\text{ atm} + P_a = 0.200\text{ atm} + 1.00\text{ atm} = 1.20\text{ atm}$, and state 2 is that at $P_2 = 1.00\text{ atm}$. Now, we can calculate the internal energy of the system in state 2 using the equation $U = \dfrac{3}{2}NkT$, since helium is a monatomic gas:

$$U_2 = \frac{3}{2}N_2kT = \frac{3}{2}\left(\frac{P_2V}{kT}\right)kT = \frac{3}{2}P_2V$$
$$= \frac{3}{2}\left(1\text{ atm}\times\frac{1.013\times10^5\text{ N/m}^2}{1\text{ atm}}\right)\left(10.0\text{ L}\times\frac{10^{-3}\text{ m}^3}{1\text{ L}}\right) = 1.52\times10^3\text{ J}$$

Next, we can use the ideal gas law, in combination with the equation $U = \dfrac{3}{2}NkT$ to get an expression for U_1:

$$\frac{U_1}{U_2} = \frac{3/2N_1kT}{3/2N_2kT} = \frac{N_1}{N_2} = \frac{P_1V/kT}{P_2V/kT} = \frac{P_1}{P_2}\text{, so that}$$
$$U_1 = \left(\frac{P_1}{P_2}\right)U_2 = \left(\frac{1.20\text{ atm}}{1.00\text{ atm}}\right)\left(1.52\times10^3\text{ J}\right) = 1.82\times10^3\text{ J}$$

and so the internal energy inside the balloon is:

$U_1 - U_2 = 1.82\times10^3\ \text{J} - 1.52\times10^3\ \text{J} = \underline{300\ \text{J}}$, greater than it would be at zero gauge pressure.

15.3 INTRODUCTION TO THE SECOND LAW OF THERMODYNAMICS: HEAT ENGINES AND THEIR EFFICIENCY

21. *With* $2.56\times10^6\ \text{J}$ *of heat transfer into this engine, a given cyclical heat engine can do only* $1.50\times10^5\ \text{J}$ *of work. (a) What is the engine's efficiency? (b) How much heat transfer to the environment takes place?*

Solution (a) The efficiency is the work out divided by the heat in:

$$Eff = \frac{W}{Q_\text{h}} = \frac{1.50\times10^5\ \text{J}}{2.56\times10^6\ \text{J}} = 0.0586,\ \text{or}\ \underline{5.86\%}$$

(b) The work output is the difference between the heat input and the wasted heat, so from the first law of thermodynamics:

$$W = Q_\text{h} - Q_\text{c} \Rightarrow Q_\text{c} = Q_\text{h} - W = 2.56\times10^6\ \text{J} - 1.50\times10^5\ \text{J} = \underline{2.41\times10^6\ \text{J}}$$

15.5 APPLICATIONS OF THERMODYNAMICS: HEAT PUMPS AND REFRIGERATORS

42. *(a) What is the best coefficient of performance for a refrigerator that cools an environment at* -30.0°C *and has heat transfer to another environment at* 45.0°C*? (b) How much work in joules must be done for a heat transfer of 4186 kJ from the cold environment? (c) What is the cost of doing this if the work costs 10.0 cents per* $3.60\times10^6\ \text{J}$ *(a kilowatt-hour)? (d) How many kJ of heat transfer occurs into the warm environment? (e) Discuss what type of refrigerator might operate between these temperatures.*

Solution (a) Using the equation $COP_{\text{ref}} = \frac{Q_c}{W} = COP_{\text{hp}} - 1$, and for the best coefficient of performance, that means make the Carnot substitution, remembering to use the absolute temperatures:

$$COP_{\text{ref}} = \frac{Q_c}{W} = COP_{\text{hp}} - 1 = \frac{1}{Eff_C} - 1 = \frac{T_h}{T_h - T_c} - 1$$
$$= \frac{T_h - (T_h - T_c)}{T_h - T_c} = \frac{T_c}{T_h - T_c} = \frac{243\text{ K}}{318\text{ K} - 243\text{ K}} = \underline{3.24}$$

(b) Using $COP_{\text{ref}} = \frac{Q_C}{W}$, again, and solve for the work done given $Q_c = 1000\text{ kcal}$:

$$COP_{\text{ref}} = \frac{Q_C}{W} \Rightarrow W = \frac{Q_C}{COP_{\text{ref}}} = \frac{1000\text{ kcal}}{3.24} = 308.6\text{ kcal} = \underline{309\text{ kcal}}$$

(c) The cost is found by converting the units of energy into units of cents:

$$\text{cost} = (308.6\text{ kcal})\left(\frac{4186\text{ J}}{1\text{ kcal}}\right)\left(\frac{10.0¢}{3.60\times10^6\text{ J}}\right) = \underline{3.59¢}$$

(d) We want to determine Q_h, so using $W = Q_h - Q_c$ gives:

$$W = Q_h - Q_c \Rightarrow$$
$$Q_h = W + Q_c = 309\text{ kcal} + 1000\text{ kcal} = 1309\text{ kcal} = 1309\text{ kcal}\times\left(\frac{4.186\text{ kJ}}{1\text{ kcal}}\right) = \underline{5479\text{ kJ}}$$

(e) The inside of this refrigerator (actually freezer) is at $-22°\text{F}\,(-30.0°\text{C})$, so this probably is a commercial meat packing freezer. The exhaust is generally vented to the outside, so as to not heat the building too much.

15.6 ENTROPY AND THE SECOND LAW OF THERMODYNAMICS: DISORDER AND THE UNAVAILABILITY OF ENERGY

47. *(a) On a winter day, a certain house loses* $5.00\times10^8\text{ J}$ *of heat to the outside (about 500,000 Btu). What is the total change in entropy due to this heat transfer alone, assuming an average indoor temperature of* $21.0°\text{C}$ *and an average outdoor*

temperature of $5.00°C$*? (b) This large change in entropy implies a large amount of energy has become unavailable to do work. Where do we find more energy when such energy is lost to us?*

Solution (a) Use $\Delta S = \frac{Q}{T}$ to calculate the change in entropy, remembering to use temperatures in Kelvin:

$$\Delta S = \frac{-Q_\text{h}}{T_\text{h}} + \frac{Q_\text{c}}{T_\text{c}} = Q\left(\frac{1}{T_\text{c}} - \frac{1}{T_\text{h}}\right) = \left(5.00\times 10^8 \text{ J}\right)\left(\frac{1}{278\text{ K}} - \frac{1}{294\text{ K}}\right) = \underline{9.78\times 10^4 \text{ J/K}}$$

(b) In order to gain more energy, we must generate it from things within the house, like a heat pump, human bodies, and other appliances. As you know, we use a lot of energy to keep our houses warm in the winter, because of the loss of heat to the outside.

53. *What is the decrease in entropy of 25.0 g of water that condenses on a bathroom mirror at a temperature of* $35.0°C$*, assuming no change in temperature and given the latent heat of vaporization to be 2450 kJ/kg?*

Solution When water condenses, it should seem reasonable that its entropy decreases, since the water gets ordered, so

$$\Delta S = \frac{Q}{T} = \frac{-mL_v}{T} = \frac{-(25.0\times 10^{-3}\text{ kg})(2450\times 10^3 \text{ J/kg})}{308\text{ K}} = \underline{-199 \text{ J/K}}$$

The entropy of the water decreases by 199 J/K when it condenses.

15.7 STATISTICAL INTERPRETATION OF ENTROPY AND THE SECOND LAW OF THERMODYNAMICS: THE UNDERLYING EXPLANATION

59. *(a) If tossing 100 coins, how many ways (microstates) are there to get the three most likely macrostates of 49 heads and 51 tails, 50 heads and 50 tails, and 51 heads and*

49 tails? (b) What percent of the total possibilities is this? (Consult Table 15.4.)

Solution (a) From Table 15.4, we can tabulate the number of ways of getting the three most likely microstates:

H	T	No. of ways
49	51	9.9×10^{28}
50	50	1.0×10^{29}
51	49	9.9×10^{28}

Total = 2.98×10^{29} = $\underline{3.0\times10^{29}}$

(b) The total number of ways is 1.27×10^{30}, so the percent represented by the three most likely microstates is:

$$\% = \frac{\text{total \# of ways to get 3 macrostates}}{\text{total \# of ways}} = \frac{3.0\times10^{29}}{1.27\times10^{30}} = 0.236 = \underline{24\%}$$

CHAPTER 16: OSCILLATORY MOTION AND WAVES

16.1 HOOKE'S LAW: STRESS AND STRAIN REVISITED

2. *It is weigh-in time for the local under-85-kg rugby team. The bathroom scale used to assess eligibility can be described by Hooke's law and is depressed 0.75 cm by its maximum load of 120 kg. (a) What is the spring's effective spring constant? (b) A player stands on the scales and depresses it by 0.48 cm. Is he eligible to play on this under-85-kg team?*

Solution (a) Using the equation $F = -kx$, where $m = 120\text{ kg}$ and $x = 0.750\times10^{-2}\text{ m}$ gives:

$$k = -\frac{F}{x} = -\frac{mg}{x} = -\frac{(120\text{ kg})(9.80\text{ m/s}^2)}{-0.750\times10^{-2}\text{ m}} = \underline{1.57\times10^5\text{ N/m}}$$

The force constant must be a positive number.

(b) $F = -kx$

$$mg = -kx \Rightarrow m = \frac{-kx}{g} = \frac{-(1.57\times10^5\text{ N/m})(0.0048\text{ m})}{-9.80\text{ m/s}^2} = 76.90\text{ kg} = \underline{77\text{ kg}}$$

<u>Yes, he is eligible to play</u>.

5. *When an 80.0-kg man stands on a pogo stick, the spring is compressed 0.120 m. (a) What is the force constant of the spring? (b) Will the spring be compressed more when he hops down the road?*

Solution (a) Using the equation $F = -kx$, where $m = 80\text{ kg}$ and $x = -0.120\text{ m}$ gives:

$$k = -\frac{F}{x} = -\frac{mg}{x} = -\frac{(80.0\text{ kg})(9.80\text{ m/s}^2)}{-0.120\text{ m}} = \underline{6.53\times10^3\text{ N/m}}$$

(b) Yes, when the man is at his lowest point in his hopping the spring will be compressed the most.

16.2 PERIOD AND FREQUENCY IN OSCILLATIONS

7. *What is the period of* 60.0 Hz *electrical power?*

Solution

Using the equation $f=\frac{1}{T}$ where $f=60.0\text{ Hz}=60.0\text{ s}^{-1}$ gives:

$$T=\frac{1}{f}=\frac{1}{60.0\text{ Hz}}=1.67\times10^{-2}\text{ s}=\underline{16.7\text{ ms}}$$

16.3 SIMPLE HARMONIC MOTION: A SPECIAL PERIODIC MOTION

15. *A 0.500-kg mass suspended from a spring oscillates with a period of 1.50 s. How much mass must be added to the object to change the period to 2.00 s?*

Solution

Using the equation $T=2\pi\sqrt{\frac{m}{k}}$ for each mass: $T_1=2\pi\sqrt{\frac{m_1}{k}};T_2=2\pi\sqrt{\frac{m_2}{k}}$,

So that the ratio of the periods can be written in terms of their masses:

$$\frac{T_1}{T_2}=\sqrt{\frac{m_1}{m_2}}\Rightarrow m_2=\left(\frac{T_1}{T_2}\right)^2 m_1=\left(\frac{2.00\text{ s}}{1.50\text{ s}}\right)^2(0.500\text{ kg})$$

The final mass minus the initial mass gives the mass that must be added:

$$\Delta m=m_2-m_1=0.889\text{ kg}-0.500\text{ kg}=\underline{0.389\text{ kg}}$$

16.4 THE SIMPLE PENDULUM

24. *What is the period of a 1.00-m-long pendulum?*

Solution

Use the equation $T = 2\pi\sqrt{\frac{L}{g}}$, where $L = 1.00\text{ m}$:

$$T = 2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{1.00\text{ m}}{9.80\text{ m/s}^2}} = \underline{2.01\text{ s}}$$

30. *(a) What is the effect on the period of a pendulum if you double its length? (b) What is the effect on the period of a pendulum if you decrease its length by 5.00%?*

Solution

(a) Use the following equation $T = 2\pi\sqrt{\frac{L}{g}}$, and write the new period in terms of the old period, where $L' = 2L$:

$$T' = 2\pi\sqrt{\frac{2L}{g}} = \sqrt{2}\left(2\pi\sqrt{\frac{L}{g_2}}\right) = \sqrt{2}T.$$

$\underline{\text{The period increases by a factor}\sqrt{2}.}$

(b) This time, $L' = 0.950L$, so that $T' = 2\pi\sqrt{\frac{0.950\,L}{g}} = \sqrt{0.950}\,T = 97.5T.$

16.5 ENERGY AND THE SIMPLE HARMONIC OSCILLATOR

36. ***Engineering Application*** *Near the top of the Citigroup Center building in New York City, there is an object with mass of* $4.00\times10^5\text{ kg}$ *on springs that have adjustable force constants. Its function is to dampen wind-driven oscillations of the building by oscillating at the same frequency as the building is being driven—the driving force is transferred to the object, which oscillates instead of the entire building. (a) What effective force constant should the springs have to make the object oscillate with a period of 2.00 s? (b) What energy is stored in the springs for a 2.00-m displacement*

from equilibrium?

Solution (a) Using the equation $T = 2\pi\sqrt{\frac{m}{k}}$, where $m = 4.00\times10^{6}\ \text{kg}$ and $T = 2.00\ \text{s}$, gives:

$$k = \frac{4\pi^2 m}{T^2} = \frac{4\pi^2(4.00\times10^5\ \text{kg})}{(2.00\ \text{s})^2} = \underline{3.95\times10^6\ \text{N/m}}$$

(b) Using the equation $\text{PE}_{\text{el}} = \frac{1}{2}kx^2$, where $x = 2.00\ \text{m}$, gives:

$$\text{PE}_{\text{el}} = \frac{1}{2}kx^2 = \frac{1}{2}(3.95\times10^6\ \text{N/m})(2.00\ \text{m})^2 = \underline{7.90\times10^6\ \text{J}}$$

16.6 UNIFORM CIRCULAR MOTION AND SIMPLE HARMONIC MOTION

37. *(a) What is the maximum velocity of an 85.0-kg person bouncing on a bathroom scale having a force constant of* $1.50\times10^6\ \text{N/m}$*, if the amplitude of the bounce is 0.200 cm? (b) What is the maximum energy stored in the spring?*

Solution (a) Use the equation $v_{\max} = \sqrt{\frac{k}{m}}X$, since the person bounces in simple harmonic motion, where $X = 0.200\times10^{-2}\ \text{m}$:

$$v_{\max} = \sqrt{\frac{k}{m}}X = \sqrt{\frac{1.50\times10^6\ \text{N/m}}{85.0\ \text{kg}}}(0.200\times10^{-2}\ \text{m}) = \underline{0.266\ \text{m/s}}$$

(b) $\text{PE} = \frac{1}{2}kX^2 = \frac{1}{2}\left(1.50\times10^6\ \text{N/m}\right)\left(0.200\times10^{-2}\ \text{m}\right)^2 = \underline{3.00\ \text{J}}$

16.8 FORCED OSCILLATIONS AND RESONANCE

45. *Suppose you have a 0.750- kg object on a horizontal surface connected to a spring that has a force constant of 150 N/m. There is simple friction between the object and surface with a static coefficient of friction* $\mu_\text{s} = 0.100$*. (a) How far can the spring be stretched without moving the mass? (b) If the object is set into oscillation with an amplitude twice the distance found in part (a), and the kinetic coefficient of friction is* $\mu_\text{k} = 0.0850$*, what total distance does it travel before stopping? Assume it starts at the maximum amplitude.*

Solution (a) We know that there are two forces acting on the mass in the horizontal direction: $\text{net } F = \mu_\text{s} N - kx = 0,$ where x is the distance the spring is stretched. We know that $\mu_\text{s} = 0.100,\ m = 0.750\text{ kg},\ k = 150\text{ N/m},$ and since the mass is on a horizontal surface, $N = mg$. We can then solve for x:

$$x = \frac{\mu_\text{s} mg}{k} = \frac{(0.100)(0.750\text{ kg})(9.80\text{ m/s}^2)}{150\text{ N/m}} = \underline{4.90 \times 10^{-3}\text{ m}}$$

So, the maximum distance the spring can be stretched without moving the mass is $\underline{4.90 \times 10^{-3}\text{ m}}$.

(b) From Example 16.7,

$$d = \frac{(1/2)k(2x)^2}{\mu_\text{k} mg} = \frac{0.5(150\text{ N/m})(9.80 \times 10^{-3}\text{ m})^2}{(0.0850)(0.750\text{ kg})(9.80\text{ m/s}^2)} = \underline{1.15 \times 10^{-2}\text{ m}}$$

16.9 WAVES

50. *How many times a minute does a boat bob up and down on ocean waves that have a wavelength of 40.0 m and a propagation speed of 5.00 m/s?*

Solution Use the equation $v_\text{w} = f\lambda$:

$$v_w = f\lambda \Rightarrow f = \frac{v_w}{\lambda} = \frac{5.00\text{ m/s}}{40.0\text{ m}} = \underline{0.125\text{ Hz}}$$

Now that we know the frequency, we can calculate the number of oscillation:

$$N = ft = (0.125\text{ Hz})(60.0\text{ s}) = \underline{7.50\text{ times}}$$

55. *Your ear is capable of differentiating sounds that arrive at the ear just 1.00 ms apart. What is the minimum distance between two speakers that produce sounds that arrive at noticeably different times on a day when the speed of sound is 340 m/s?*

Solution Use the definition of velocity, $v = \frac{d}{t}$, given the wave velocity and the time:

$$d = v_w t = (340\text{ m/s})(1.00 \times 10^{-3}\text{ s}) = 0.340\text{ m} = \underline{34.0\text{ cm}}$$

Therefore, objects that are 34.0 cm apart (or farther) will produce sounds that the human ear can distinguish.

16.10 SUPERPOSITION AND INTERFERENCE

62. *Three adjacent keys on a piano (F, F-sharp, and G) are struck simultaneously, producing frequencies of 349, 370, and 392 Hz. What beat frequencies are produced by this discordant combination?*

Solution There will be three different beat frequencies because of the interactions between the three different frequencies.

Using the equation $f_B = |f_1 - f_2|$ gives:

$$f_{B1} = |370\text{ Hz} - 349\text{ Hz}| = \underline{21\text{ Hz}},$$
$$f_{B2} = |392\text{ Hz} - 370\text{ Hz}| = \underline{22\text{ Hz}}$$
$$f_{B3} = |392\text{ Hz} - 349\text{ Hz}| = \underline{43\text{ Hz}}$$

16.11 ENERGY IN WAVES: INTENSITY

71. ***Medical Application*** *(a) What is the intensity in* W/m^2 *of a laser beam used to burn away cancerous tissue that, when 90.0% absorbed, puts 500 J of energy into a circular spot 2.00 mm in diameter in 4.00 s? (b) Discuss how this intensity compares to the average intensity of sunlight (about* 1 W/m^2*) and the implications that would have if the laser beam entered your eye. Note how your answer depends on the time duration of the exposure.*

Solution (a) Using the equations $I = \dfrac{P}{A}$ and $P = \dfrac{W}{t}$, we see that

$$W = Pt = (0.900IA)t = 0.900I\pi r^2 t,\ \text{so that:}$$
$$I = \frac{W}{0.900\pi r^2 t} = \frac{500\text{ J}}{0.900\pi\left(2.00\times10^{-3}\text{ m}\right)^2(4.00\text{ s})} = \underline{1.11\times10^{7}\text{ W/m}^2}$$

(b) The intensity of a laser is about 10^4 times that of the sun, so clearly lasers can be very damaging if they enter your eye! This means that starting into a laser for one second is equivalent to starting at the sun for 11 hours without blinking!

CHAPTER 17: PHYSICS OF HEARING

17.2 SPEED OF SOUND, FREQUENCY, AND WAVELENGTH

1. *When poked by a spear, an operatic soprano lets out a 1200-Hz shriek. What is its wavelength if the speed of sound is 345 m/s?*

Solution Use the equation $v_\text{w} = f\lambda$, where $f = 1200\ \text{Hz}$ and $v_\text{w} = 345\ \text{m/s}$:

$$\lambda = \frac{v_\text{w}}{f} = \frac{345\ \text{m/s}}{1200\ \text{Hz}} = \underline{0.288\ \text{m}}$$

7. *Dolphins make sounds in air and water. What is the ratio of the wavelength of a sound in air to its wavelength in seawater? Assume air temperature is* $20.0°\text{C}$.

Solution The wavelengths of sounds in air and water are different because the speed of sound is different in air and water. We know $v_\text{seawater} = 1540\ \text{m/s}$ (from Table 17.1) and $v_\text{air} = 343\ \text{m/s}$ at $20.0°\text{C}$ from Problem 17.5, so from the equation $v_\text{w} = f\lambda$ we know $v_\text{seawater} = f\lambda_\text{seawater}$ and $v_\text{air} = f\lambda_\text{air}$, so we can determine the ratio of the wavelengths:

$$\frac{v_\text{air}}{v_\text{seawater}} = \frac{\lambda_\text{air}}{\lambda_\text{seawater}} \Rightarrow \frac{\lambda_\text{air}}{\lambda_\text{seawater}} = \frac{343\ \text{m/s}}{1540\ \text{m/s}} = \underline{0.223}$$

17.3 SOUND INTENSITY AND SOUND LEVEL

13. *The warning tag on a lawn mower states that it produces noise at a level of 91.0 dB. What is this in watts per meter squared?*

Solution $\beta = 10\log_{10}\left(\dfrac{I}{I_0}\right)$, where $I_0 = 10^{-12}\ \text{W/m}^2$, so that $I = I_0 10^{\beta/10}$, and

$$I = (1.00\times10^{-12}\ \text{W/m}^2)10^{91.0/10.0} = \underline{1.26\times10^{-3}\ \text{W/m}^2}$$

(To calculate an exponent that is not an integer, use the x^y-key on your calculator.)

21. *People with good hearing can perceive sounds as low in level as* $-8.00\ \text{dB}$ *at a frequency of 3000 Hz. What is the intensity of this sound in watts per meter squared?*

Solution $\beta = 10\log\left(\dfrac{I}{I_0}\right)$, so that

$$I = I_0 10^{\beta/10} = (1.00\times10^{-12}\ \text{W/m}^2)10^{-8.00/10.0} = \underline{1.58\times10^{-13}\ \text{W/m}^2}$$

27. *(a) Ear trumpets were never very common, but they did aid people with hearing losses by gathering sound over a large area and concentrating it on the smaller area of the eardrum. What decibel increase does an ear trumpet produce if its sound gathering area is* $900\ \text{cm}^2$ *and the area of the eardrum is* $0.500\ \text{cm}^2$*, but the trumpet only has an efficiency of 5.00% in transmitting the sound to the eardrum? (b) Comment on the usefulness of the decibel increase found in part (a).*

Solution (a) Using the equation $I = \dfrac{P}{A}$, we see that for the same power, $\dfrac{I_2}{I_1} = \dfrac{A_1}{A_2}$, so for a 5.00% efficiency: $\dfrac{I_e}{I_1} = \dfrac{A_1}{A_e} = \dfrac{(0.0500)(900\ \text{cm}^2)}{0.500\ \text{cm}^2} = 90$.

Now, using the equation $\beta(\text{dB}) = 10\log_{10}\left(\dfrac{I}{I_0}\right)$, and remembering that $\log A - \log B = \log\dfrac{A}{B}$, we see that:

$$\beta_e - \beta_t = 10\log\left(\frac{I_e}{I_0}\right) - 10\log\left(\frac{I_t}{I_0}\right) = 10\log\left(\frac{I_e}{I_t}\right) = 10\log(90) = 19.54\text{ dB} = \underline{19.5\text{ dB}}$$

(b) This increase of approximately 20 dB increases the sound of a normal conversation to that of a loud radio or classroom lecture (see Table 17.2). For someone who cannot hear at all, this will not be helpful, but for someone who is starting to lose their ability to hear, it may help. Unfortunately, ear trumpets are very cumbersome, so even though they could help, the inconvenience makes them rather impractical.

17.4 DOPPLER EFFECT AND SONIC BOOMS

33. *A spectator at a parade receives an 888-Hz tone from an oncoming trumpeter who is playing an 880-Hz note. At what speed is the musician approaching if the speed of sound is 338 m/s?*

Solution We can use the equation $f_{obs} = f_s \dfrac{v_w}{v_w - v_s}$ (with the minus sign because the source is approaching to determine the speed of the musician (the source), given $f_{obs} = 888\text{ Hz}, f_s = 880\text{ Hz}, \text{and } v_w = 338\text{ m/s}$:

$$v_s = \frac{v_w(f_{obs} - f_s)}{f_{obs}} = \frac{(338\text{ m/s})(888\text{ Hz} - 880\text{ Hz})}{888\text{ Hz}} = \underline{3.05\text{ m/s}}$$

17.5 SOUND INTERFERENCE AND RESONANCE: STANDING WAVES IN AIR COLUMNS

39. *What beat frequencies will be present: (a) If the musical notes A and C are played together (frequencies of 220 and 264 Hz)? (b) If D and F are played together (frequencies of 297 and 352 Hz)? (c) If all four are played together?*

Solution (a) Using the equation $f_{\mathrm{B}}=|f_1-f_2|$: $f_{\mathrm{B,A\&C}}=|f_1-f_2|=|264\ \mathrm{Hz}-220\ \mathrm{Hz}|=\underline{44\ \mathrm{Hz}}$

(b) $f_{\mathrm{B,D\&F}}=|f_1-f_2|=|352\ \mathrm{Hz}-297\ \mathrm{Hz}|=\underline{55\ \mathrm{Hz}}$

(c) We get beats from every combination of frequencies, so in addition to the two beats above, we also have:

$$f_{\mathrm{B,F\&A}}=352\ \mathrm{Hz}-220\ \mathrm{Hz}=\underline{132\ \mathrm{Hz}};$$
$$f_{\mathrm{B,F\&C}}=352\ \mathrm{Hz}-264\ \mathrm{Hz}=\underline{88\ \mathrm{Hz}};$$
$$f_{\mathrm{B,D\&C}}=297\ \mathrm{Hz}-264\ \mathrm{Hz}=\underline{33\ \mathrm{Hz}};$$
$$f_{\mathrm{B,D\&A}}=297\ \mathrm{Hz}-220\ \mathrm{Hz}=\underline{77\ \mathrm{Hz}}$$

45. *How long must a flute be in order to have a fundamental frequency of 262 Hz (this frequency corresponds to middle C on the evenly tempered chromatic scale) on a day when air temperature is* $20.0^\circ\mathrm{C}$*? It is open at both ends.*

Solution

We know that the frequency for a tube open at both ends is:

$$f_n=n\left(\frac{v}{2L}\right)\ \text{for}\ n=1,2,3\ldots$$

If the fundamental frequency $(n=1)$ is $f_1=262\ \mathrm{Hz}$, we can determine the length:

$f_1=\dfrac{v_{\mathrm{w}}}{2L}\Rightarrow L=\dfrac{v_{\mathrm{w}}}{2f_1}$. We need to determine the speed of sound, from the equation

$v_{\mathrm{w}}=(331\ \mathrm{m/s})\sqrt{\dfrac{T(\mathrm{K})}{273\ \mathrm{K}}}$, since we are told the air temperature:

$$v_{\mathrm{w}}=(331\ \mathrm{m/s})\sqrt{\frac{T\ (\mathrm{K})}{273\ \mathrm{K}}}=(331\ \mathrm{m/s})\sqrt{\frac{293\ \mathrm{K}}{273\ \mathrm{K}}}=342.9\ \mathrm{m/s}.$$

Therefore, $L=\dfrac{342.9\ \mathrm{m/s}}{2(262\ \mathrm{Hz})}=0.654\ \mathrm{m}=\underline{65.4\ \mathrm{cm}}.$

51. *Calculate the first overtone in an ear canal, which resonates like a 2.40-cm-long tube closed at one end, by taking air temperature to be* $37.0°\text{C}$*. Is the ear particularly sensitive to such a frequency? (The resonances of the ear canal are complicated by its nonuniform shape, which we shall ignore.)*

Solution

First, we need to determine the speed of sound at $37.0°\text{C}$, using the equation

$$v_\text{w} = (331\ \text{m/s})\sqrt{\frac{T\,(\text{K})}{273\ \text{K}}} = (331\ \text{m/s})\sqrt{\frac{310\ \text{K}}{273\ \text{K}}} = 352.7\ \text{m/s}.$$

Next, for tubes closed at one end: $f_n = n\left(\frac{v_\text{w}}{4L}\right)$ for $n = 1,3,5...$, we can determine the frequency of the first overtone $(n=3)$

$$f_3 = 3\frac{352.7\ \text{m/s}}{4(0.0240\ \text{m})} = 1.10\times10^4\ \text{Hz} = \underline{11.0\ \text{kHz}}.$$

The ear is not particularly sensitive to this frequency, so we don't hear overtones due to the ear canal.

17.6 HEARING

57. *What are the closest frequencies to 500 Hz that an average person can clearly distinguish as being different in frequency from 500 Hz? The sounds are not present simultaneously.*

Solution We know that we can discriminate between two sounds if their frequencies differ by at least 0.3%, so the closest frequencies to 500 Hz that we can distinguish are $f = (500\ \text{Hz})(1\pm0.003) = \underline{498.5\ \text{Hz}}$ and $\underline{501.5\ \text{Hz}}$.

63. *What is the approximate sound intensity level in decibels of a 600-Hz tone if it has a loudness of 20 phons? If it has a loudness of 70 phons?*

Solution From Figure 17.36: a 600 Hz tone at a loudness of 20 phons has a sound level of about 23 dB, while a 600 Hz tone at a loudness of 70 phons has a sound level of about 70 dB.

69. *A person has a hearing threshold 10 dB above normal at 100 Hz and 50 dB above normal at 4000 Hz. How much more intense must a 100-Hz tone be than a 4000-Hz tone if they are both barely audible to this person?*

Solution From Figure 17.36, the 0 phons line is normal hearing. So, this person can barely hear a 100 Hz sound at 10 dB above normal, requiring a 47 dB sound level (β_1). For a 4000 Hz sound, this person requires 50 dB above normal, or a 43 dB sound level (β_2) to be audible. So, the 100 Hz tone must be 4 dB higher than the 4000 Hz sound. To calculate the difference in intensity, use the equation $\beta_1 - \beta_2 = 10\log\left(\frac{I_1}{I_0}\right) - 10\log\left(\frac{I_2}{I_0}\right) = 10\log\left(\frac{I_1}{I_2}\right)$ and convert the difference in decibels to a ratio of intensities. Substituting in the values from above gives:

$$10\log\left(\frac{I_1}{I_2}\right) = 47\ \text{dB} - 43\ \text{dB} = 4\ \text{dB},\ \text{or}\ \frac{I_1}{I_2} = 10^{4/10} = \underline{2.5}$$

So the 100 Hz tone must be 2.5 times more intense than the 4000 Hz sound to be audible by this person.

17.7 ULTRASOUND

77. *(a) Calculate the minimum frequency of ultrasound that will allow you to see details as small as 0.250 mm in human tissue. (b) What is the effective depth to which this sound is effective as a diagnostic probe?*

Solution (a) From Table 17.1, the speed of sound in tissue is $v_w = 1540\ \text{m/s}$, so using $v_w = f\lambda$, we find the minimum frequency to resolve 0.250 mm details is:

$$f = \frac{v_w}{\lambda} = \frac{1540\ \text{m/s}}{0.250\times10^{-3}\ \text{m}} = \underline{6.16\times10^{6}\ \text{Hz}}$$

(b) We know that the accepted rule of thumb is that you can effectively scan to a depth of about 500λ into tissue, so the effective scan depth is:

$$500\lambda = 500\left(0.250\times10^{-3}\ \text{m}\right) = 0.125\ \text{m} = \underline{12.5\ \text{cm}}$$

83. *A diagnostic ultrasound echo is reflected from moving blood and returns with a frequency 500 Hz higher than its original 2.00 MHz. What is the velocity of the blood? (Assume that the frequency of 2.00 MHz is accurate to seven significant figures and 500 Hz is accurate to three significant figures.)*

Solution This problem requires two steps: (1) determine the frequency the blood receives (which is the frequency that is reflected), then (2) determine the frequency that the scanner receives. At first, the blood is like a moving observer, and the equation $f_{obs} = f_s\left(\frac{v_w + v_s}{v_w}\right)$ gives the frequency it receives (with the plus sign used because the blood is approaching): $f_b = f_s\left(\frac{v_w + v_b}{v_w}\right)$ (where v_b = blood velocity). Next, this frequency is reflected from the blood, which now acts as a moving source. The equation $f_{obs} = f_s\left(\frac{v_w}{v_w \mp v_s}\right)$ (with the minus sign used because the blood is still approaching) gives the frequency received by the scanner:

$$f'_{obs} = f_b\left(\frac{v_w}{v_w - v_b}\right) = f_s\left(\frac{v_w + v_b}{v_w}\right)\left(\frac{v_w}{v_w - v_b}\right) = f_s\left(\frac{v_w + v_b}{v_w - v_b}\right)$$

Solving for the speed of blood gives:

$$v_b = v_w\left(\frac{f'_{obs} - f_s}{f'_{obs} + f_s}\right) = \frac{(1540\ \text{m/s})(500\ \text{Hz})}{(2.00\times10^{6}\ \text{Hz} + 500\ \text{Hz}) + 2.00\times10^{6}\ \text{Hz}} = \underline{0.192\ \text{m/s}}$$

The blood's speed is 19.2 cm/s.

CHAPTER 18: ELECTRIC CHARGE AND ELECTRIC FIELD

18.1 STATIC ELECTRICITY AND CHARGE: CONSERVATION OF CHARGE

1. *Common static electricity involves charges ranging from nanocoulombs to microcoulombs. (a) How many electrons are needed to form a charge of –2.00 nC? (b) How many electrons must be removed from a neutral object to leave a net charge of* $0.500\ \mu\text{C}$*?*

Solution (a) Since one electron has charge of $q_e = -1.6\times10^{-19}\ \text{C}$, we can determine the number of electrons necessary to generate a total charge of -2.00 nC by using the equation $Nq_e = Q$, so that $N = \frac{Q}{q_e} = \frac{-2.00\times10^{-9}\ \text{C}}{-1.60\times10^{-19}\ \text{C}} = \underline{1.25\times10^{10}\ \text{electrons}}$.

(b) Similarly we can determine the number of electrons removed from a neutral object to leave a charge of $0.500\ \mu\text{C}$ of charge so that:

$$N = \frac{-0.500\times10^{-6}\ \text{C}}{-1.60\times10^{-19}\ \text{C}} = \underline{3.13\times10^{12}\ \text{electrons}}$$

18.2 CONDUCTORS AND INSULATORS

7. *A 50.0 g ball of copper has a net charge of* $2.00\ \mu\text{C}$*. What fraction of the copper's electrons has been removed? (Each copper atom has 29 protons, and copper has an atomic mass of 63.5.)*

Solution Recall that Avogadro's number is $N_A = 6.02\times10^{23}$ atoms/mole. Now we need to determine the number of moles of copper that are present. We do this using the mass and the atomic mass: $n = \frac{m}{A} = \frac{50.0\ \text{g}}{63.5\ \text{g/mol}}$

So since there are 29 protons per atom we can determine the number of protons,

N_p, from:

$$N_p = nN_A \times 29 \text{ protons/atom}$$
$$= \left(\frac{50.0 \text{ gm}}{63.5 \text{ g/mol}}\right)\left(\frac{6.02\times10^{23} \text{ atoms}}{\text{mol}}\right)\times\frac{29 \text{ protons}}{\text{cu atom}} = 1.375\times10^{25} \text{ protons}$$

Since there is the same number of electrons as protons in a neutral atom, before we remove the electrons to give the copper a net charge, we have 1.375×10^{25} electrons.

Next we need to determine the number of electrons we removed to leave a net charge of $2.00\ \mu\text{C}$. We need to remove $-2.00\ \mu\text{C}$ of charge, so the number of electrons to be removed is given by

$$\text{N}_{\text{e,removed}} = \frac{Q}{q_e} = \frac{-2.00\times10^{-6}\text{ C}}{-1.60\times10^{-19}\text{ C}} = 1.25\times10^{13} \text{ electrons removed.}$$

Finally we can calculate the fraction of copper's electron by taking the ratio of the number of electrons removed to the number of electrons originally present:

$$\frac{N_{\text{e,removed}}}{N_{\text{e,initially}}} = \frac{1.25\times10^{13}}{1.375\times10^{25}} = \underline{9.09\times10^{-13}}$$

18.3 COULOMB'S LAW

13. *Two point charges are brought closer together, increasing the force between them by a factor of 25. By what factor was their separation decreased?*

Solution

Using the equation $F_1 = k\dfrac{q_1q_2}{r_1^2}$, we see that the force is inversely proportional to the separation distance squared, so that $F_1 = K\dfrac{q_1q_2}{r_1^2}$ and $F_2 = K\dfrac{q_1q_2}{r_2^2}$

Since we know the ratio of the forces we can determine the ratio of the separation distances: $\dfrac{F_1}{F_2} = \left(\dfrac{r_2}{r_1}\right)^2$ so that $\dfrac{r_2}{r_1} = \sqrt{\dfrac{F_1}{F_2}} = \sqrt{\dfrac{1}{25}} = \dfrac{1}{5}$

The separation decreased by a factor of $\underline{5}$.

20. *(a) Common transparent tape becomes charged when pulled from a dispenser. If one piece is placed above another, the repulsive force can be great enough to support the top piece's weight. Assuming equal point charges (only an approximation), calculate the magnitude of the charge if electrostatic force is great enough to support the weight of a 10.0 mg piece of tape held 1.00 cm above another. (b) Discuss whether the magnitude of this charge is consistent with what is typical of static electricity.*

Solution (a) If the electrostatic force is to support the weight of 10.0 mg piece of tape, it must be a force equal to the gravitational force on the tape, so using the equation $F_1 = k\frac{q_1 q_2}{r_1^2}$ and the assumption that the point charges are equal, we can set electrostatic force equal to gravitational force.

$$F = k\frac{q_1 q_2}{r^2} = \frac{kq^2}{r^2} = mg \Rightarrow q = \left(\frac{r^2 mg}{k}\right)^{1/2}$$

$$= \left[\frac{(0.0100\text{ m})^2(10.0\times10^{-6}\text{ kg})(9.80\text{ m/s}^2)}{(9.00\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)}\right]^{1/2} = \underline{1.04\times10^{-9}\text{ C}}.$$

(b) This charge is approximately 1 nC, which is consistent with the magnitude of the charge of typical static electricity.

25. *Point charges of $5.00\ \mu\text{C}$ and $-3.00\ \mu\text{C}$ are placed 0.250 m apart. (a) Where can a third charge be placed so that the net force on it is zero? (b) What if both charges are positive?*

Solution (a) We know that since the negative charge is smaller, the third charge should be placed to the right of the negative charge if the net force on it to be zero. So if we want $F_{\text{net}} = F_1 + F_2 = 0$, we can use the equation $F_1 = k\frac{q_1 q_2}{r_1^2}$ to write the forces in terms of distances:

$$\frac{Kq_1 q_2}{r_1^2} + \frac{Kq_2 q}{r_2^2} = Kq\left(\frac{q_1}{r_1^2} + \frac{q_2}{r_2^2}\right) = 0,$$ or since $r_1 = 0.250\text{ m} + d$ and $\text{r}_2 = d$,

$$\frac{5\times10^{-6}C}{(0.250\text{ m}+d)^2} = \frac{3\times10^{-6}}{d^2},$$ or $d = \sqrt{\frac{3}{5}}(0.250\text{ m}) + \sqrt{\frac{3}{5}}d$ so that

$d\left(1-\sqrt{\frac{3}{5}}\right)=\frac{\sqrt{3}}{5}(0.250\text{ m})$ and finally, $d=\dfrac{\sqrt{\frac{3}{5}}(0.250\text{ m})}{1-\frac{\sqrt{3}}{5}}=\underline{0.859\text{ m}}$

The charge must be placed at a distance of 0.859 m to the far side of the negative charge.

(b) This time we know that the charge must be placed between the two positive charges and closer to the $3\ \mu\text{C}$ charge for the net force to be zero. So if we want $F_{\text{net}}=F_1+F_2=0$, we can again use $F_1=k\dfrac{q_1q_2}{r_1^{\ 2}}$ to write the forces in terms of distances: $\dfrac{Kq_1q_2}{r_1^2}+\dfrac{Kq_2q}{r_2^2}=Kq\left(\dfrac{q_1}{r_1^2}+\dfrac{q_2}{r_2^2}\right)=0$

Or since $r_1=0.250\text{m}-r_2$ $\dfrac{5\times10^{-6}\text{ C}}{(0.250-r_2)^2}=\dfrac{3\times10^{-6}}{r_2^{\ 2}}$, or

$r_2^{\ 2}=\frac{3}{5}(0.250\text{ m}-r_2)^2$, or $r_2=\sqrt{\frac{3}{5}}(0.250\text{ m}-r_2)$, and finally

$$r_2=\frac{\sqrt{\frac{3}{5}}(0.250\text{ m})}{1+\sqrt{\frac{3}{5}}}=\underline{0.109\text{ m}}$$

The charge must be placed at a distance of 0.109m from the $3\ \mu\text{C}$ charge.

18.4 ELECTRIC FIELD: CONCEPT OF A FIELD REVISITED

32. *(a) Find the direction and magnitude of an electric field that exerts a* $4.80\times10^{-17}\text{ N}$ *westward force on an electron. (b) What magnitude and direction force does this field exert on a proton?*

Solution (a) Using the equation $F=qE$ we can find the electric field caused by a given force on a given charge (taking eastward direction to be positive):

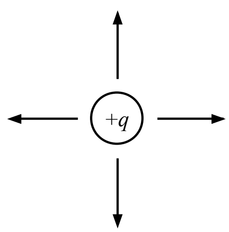
$$E=\frac{F}{q}=\frac{-4.80\times10^{-17}\text{ C}}{-1.60\times10^{-19}\text{ C}}=\underline{300\text{ N/C (east)}}$$

(b) The force should be equal to the force on the electron only in the opposite direction. Using $F = qE$ we get
$F = qE = (1.60\times10^{-19}\ \text{C})(300\ \text{N/C}) = \underline{4.80\times10^{-17}\ \text{N (east)}}$, as we expected.

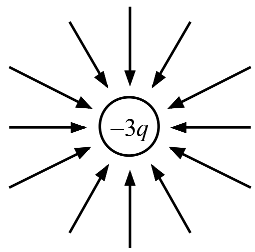
18.5 ELECTRIC FIELD LINES: MULTIPLE CHARGES

33. *(a) Sketch the electric field lines near a point charge $+q$. (b) Do the same for a point charge $-3.00q$.*

Solution (a)

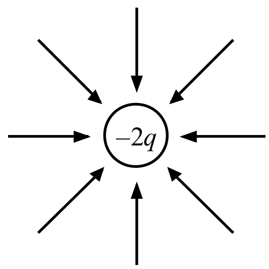


(b)

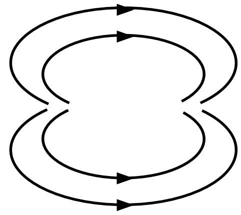


34. *Sketch the electric field lines a long distance from the charge distributions shown in Figure 18.26(a) and (b).*

Solution (a)



(b)

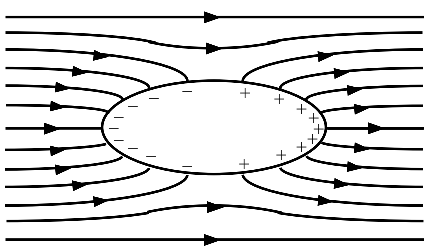


No net charge is seen from far away.

18.7 CONDUCTORS AND ELECTRIC FIELDS IN STATIC EQUILIBRIUM

37. *Sketch the electric field lines in the vicinity of the conductor in Figure 18.48, given the field was originally uniform and parallel to the object's long axis. Is the resulting field small near the long side of the object?*

Solution



The field lines deviate from their original horizontal direction because the charges within the object rearrange. The field lines will come into the object perpendicular to the surface and will leave the other side of the object perpendicular to the surface.

Yes, the field is smaller near the long side of the object. This is evident because there are fewer field lines near the long side of the object and there are more field lines near the point of the object.

44. *(a) Find the total Coulomb force on a charge of 2.00 nC located at $x = 4.00\ \text{cm}$ in Figure 18.52(b), given that $q = 1.00\ \mu\text{C}$. (b) Find the x-position at which the electric field is zero in Figure 18.52(b).*

Solution (a) According to Figure 18.52, the point charges are given by

$q_1 = -2.00\ \mu\text{C}$ at $\text{x} = 1.00\ \text{cm}$; $q_5 = +1.00\ \mu\text{C}$ at $\text{x} = 5.00\ \text{cm}$;

$q_8 = 3.00\ \mu\text{C}$ at $\text{x} = 8.00\ \text{cm}$ and $q_{14} = -1.00\ \mu\text{C}$ at $\text{x} = 14.0\ \text{cm}$

If a $2\ \text{nC}$ charge is placed at $\text{x} = 4.00\ \text{cm}$, the force it feels from other charges is found from the equation $F = \dfrac{Kq_1q}{r_1^2}$. The net force is the vector addition of the force due to each point charge, but since the point charges are all along the *x*-axis, the forces add like numbers; thus the net force is given by

$$F = \frac{Kq_1q}{r_1^2} - \frac{Kq_5q}{r_5^2} - \frac{Kq_8q}{r_8^2} - \frac{Kq_{14}q}{r_{14}^2} = Kq\left(\frac{q_1}{r_1^2} - \frac{q_5}{r_5^2} - \frac{q_8}{r_8^2} - \frac{q_{14}}{r_{14}^2}\right)$$

To the right, notice that the term involving the charge q_1 has the opposite sign because it pulls in the opposite direction than the other three charges.

Substituting in the values given:

$$F = \left(\frac{9.00\times10^9\ \text{N.m}^2}{\text{C}^2}\right)\left(2.00\times10^{-9}\ \text{C}\right)\times$$
$$\left[\frac{-2.00\times10^{-6}\ \text{C}}{(0.0400\ \text{m}-0.0100\ \text{m})^2} - \frac{1.00\times10^{-6}\ \text{C}}{(0.0500\ \text{m}-0.0400\ \text{m})^2} - \frac{3.00\times10^{-6}\ \text{C}}{(0.0800\ \text{m}-0.0400\ \text{m})^2} - \frac{-1.00\times10^{-6}\ \text{C}}{(0.140\ \text{m}-0.0400\ \text{m})^2}\right]$$
$$= \underline{-0.252\ \text{N (right) or } 0.252\ \text{N to the left}}$$

(b) The only possible location where the total electric field could be zero is between 5.00 and 8.00 cm, since in that range the closest charges create forces on the test charge in opposite directions. So that is the only region we will consider. For the total electric field to be zero between 5.00 and 8.00 cm, we know that:

$$E = \frac{Kq_1}{r_1^2} + \frac{Kq_5}{r_5^2} - \frac{Kq_8}{r_8^2} - \frac{Kq_{14}}{r_{14}^2} = 0$$

Dividing by common factors and ignoring units (but remembering *x* has a unit of cm), we can get a simplified expression:

$$\left[y = \frac{-2}{(x-1)^2} + \frac{1}{(x-5)^2} - \frac{3}{(x-8)^2} - \frac{-1}{(x-14)^2}\right]$$

We can graph this function, using a graphing calculator or graphing program, to determine the values of *x* that yield $y = 0$.

Therefore the total electric field is zero at $\underline{x = 6.07\,\text{cm}}$.

50. *(a) Find the electric field at the center of the triangular configuration of charges in Figure 18.54, given that* $q_a = +2.50\text{ nC}$*,* $q_b = 8.00\text{ nC}$*, and* $q_c = +1.50\text{ nC}$*. (b) Is there any combination of charges, other than* $q_a = q_b = q_c$*, that will produce a zero strength electric field at the center of the triangular configuration?*

Solution (a) To determine the electric field at the center, we first must determine the distance from each of the charges to the center of the triangle. Since the triangle is equilateral, the center of the triangle will be halfway across the base and 1/3 of the way up the height. To determine the height use the Pythagorean theorem, or the height is given by $h = \sqrt{(25.0\text{ cm})^2 - (12.5\text{ cm})^2} = 21.7\text{ cm}$. So the distance from each charge to the center of the triangle is 2/3 of 21.7 cm, or $r = \frac{2}{3}(21.7\text{ cm}) = 14.4\text{ cm}$. Since $E = k\frac{Q}{r^2}$,

$$\vec{E}_\text{a} = k\frac{q_\text{a}}{r^2} = \left(9.00\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(\frac{2.50\times10^{-9}\text{ C}}{(0.144\text{ m})^2}\right) = 1085\text{ N/C}$$ at a $90°$ angle below the horizontal,

$$\vec{E}_\text{b} = k\frac{q_\text{b}}{r^2} = \left(9.00\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(\frac{-8.00\times10^{-9}\text{ C}}{(0.144\text{ m})^2}\right) = 3472\text{ N/C}$$ at a $30°$ angle below the horizontal, and

$$\vec{E}_\text{c} = k\frac{q_\text{c}}{r^2} = \left(9.00\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(\frac{1.50\times10^{-9}\text{ C}}{(0.144\text{ m})^2}\right) = 681.0\text{ N/C}$$ at a $30°$ angle above the horizontal. Adding the vectors by components gives:

$$E_x = E_\text{a}\cos(-90°) + E_\text{b}\cos(-30°) + E_\text{c}\cos 30°$$
$$E_x = 0\text{ N/C} + 3472\text{ N/C}(0.860) + 681.0\text{ N/C}(0.8660) = 3597\text{ N/C}$$
$$E_y = E_\text{a}\sin(-90°) + E_\text{b}\sin(-30°) + E_\text{c}\sin 30°$$
$$E_y = -1085\text{ N/C} - 3472\text{ N/C}(0.5000) + 681\text{ N/C}(0.5000) = -2481\text{ N/C}$$

So that the electric field is given by:

$$E = \sqrt{E_x^2 + E_y^2} = \sqrt{(3597\text{ N/C})^2 + (-2481\text{ N/C})^2} = 4370\text{ N/C}$$ and

$$\theta = \tan^{-1}\frac{E_y}{E_x} = \tan^{-1}\frac{-2481\text{ N/C}}{3597\text{ N/C}} = -34.6°\text{, or } \underline{\vec{E} = 4.37\times10^3\text{ N/C}, 34.6°}$$

below the horizontal.

(b) No, there are no combinations other than $q_\text{a} = q_\text{b} = q_\text{c}$ that will produce a zero strength field at the center of the triangular configuration because of the vector nature of the electric field. Consider the two cases: (1) all charges have the same sign and (2) one charge have a different sign than the other two. For case (1), symmetry dictates that the charges must be all the same magnitude, if a test charge is not to feel a force at the center of the triangle. For case (2), a positive test charge would feel a force towards the negative charge(s) and away from the positive charge(s). Therefore there is no combination that would produce a zero strength electric field at the center of the triangle.

18.8 APPLICATIONS OF ELECTROSTATICS

56. *What can you say about two charges q_1 and q_2, if the electric field one-fourth of the way from q_1 to q_2 is zero?*

Solution If the electric field is zero 1/4 from the way of q_1 and q_2, then we know from the equation $E = k\frac{Q}{r^2}$ that $|E_1| = |E_2| \Rightarrow \frac{Kq_1}{x^2} = \frac{Kq_2}{(3x)^2}$ so that $\underline{\frac{q_2}{q_1} = \frac{(3x)^2}{x^2} = 9}$

The charge q_2 is 9 times larger than q_1.

65. ***Unreasonable Results*** *(a) Two 0.500 g raindrops in a thunderhead are 1.00 cm apart when they each acquire 1.00 mC charges. Find their acceleration. (b) What is unreasonable about this result? (c) Which premise or assumption is responsible?*

Solution (a) To determine the acceleration, use Newton's Laws and the equation $F = k\frac{q_1 q_2}{r^2}$:

$$F = ma = \frac{kq_1q_2}{r^2} \Rightarrow$$

$$a = \frac{kq^2}{mr^2} = \frac{(9.00\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.00\times10^{-3}\ \text{C})^2}{(0.500\times10^{-3}\ \text{kg})(0.0100\ \text{m})^2} = \underline{1.80\times10^{11}\ \text{m/s}^2}$$

(b) The resulting acceleration is unreasonably large; the raindrops would not stay together.

(c) The assumed charge of $1.00\ \text{mC}$ is much too great; typical static electricity is on the order of $1\ \mu\text{C}$ or less.

CHAPTER 19: ELECTRIC POTENTIAL AND ELECTRIC FIELD

19.1 ELECTRIC POTENTIAL ENERGY: POTENTIAL DIFFERENCE

6. ***Integrated Concepts** (a) What is the average power output of a heart defibrillator that dissipates 400 J of energy in 10.0 ms? (b) Considering the high-power output, why doesn't the defibrillator produce serious burns?*

Solution (a) The power is the work divided by the time, so the average power is:

$$P = \frac{W}{t} = \frac{400\text{ J}}{10.0 \times 10^{-3}\text{ s}} = \underline{4.00 \times 10^{4}\text{ W}}.$$

(b) A defibrillator does not cause serious burns because the skin conducts electricity well at high voltages, like those used in defibrillators. The gel used aids in the transfer of energy to the body, and the skin doesn't absorb the energy, but rather, lets it pass through to the heart.

19.2 ELECTRIC POTENTIAL IN A UNIFORM ELECTRIC FIELD

17. *(a) Will the electric field strength between two parallel conducting plates exceed the breakdown strength for air ($3.0 \times 10^{6}\text{ V/m}$) if the plates are separated by 2.00 mm and a potential difference of $5.0 \times 10^{3}\text{ V}$ is applied? (b) How close together can the plates be with this applied voltage?*

Solution (a) Using the equation $E = \frac{V_{\text{AB}}}{d}$, we can determine the electric field strength produced between two parallel plates since we know their separation distance and the potential difference across the plates:

$$E = \frac{V_{\text{AB}}}{d} = \frac{5.0\times 10^{3}\text{ V}}{2.00\times 10^{-3}\text{ m}} = \underline{2.5\times 10^{6}\text{ V/m}} < 3\times 10^{6}\,\text{V/m}.$$

No, the field strength is smaller than the breakdown strength for air.

(b) Using the equation $E = \frac{V_{\text{AB}}}{d}$, we can now solve for the separation distance, given the potential difference and the maximum electric field strength:

$$d = \frac{V_{\text{AB}}}{E} = \frac{5.0\times 10^{3}\text{ V}}{3.0\times 10^{6}\text{ V/m}} = 1.67\ \times 10^{-3}\text{ m} = \underline{1.7\,\text{mm}}.$$

So, the plates must not be closer than 1.7 mm to avoid exceeding the breakdown strength of air.

23. *An electron is to be accelerated in a uniform electric field having a strength of* $2.00\times 10^{6}\text{ V/m}$*. (a) What energy in keV is given to the electron if it is accelerated through 0.400 m? (b) Over what distance would it have to be accelerated to increase its energy by 50.0 GeV?*

Solution

(a) Using the equation $\Delta\text{KE} = q\Delta V$, we can get an expression for the change in energy terms of the potential difference and its charge. Also, we know from the equation $E = \frac{V_{\text{AB}}}{d}$ that we can express the potential difference in the terms of the electric field strength and the distance traveled, so that:

$$\begin{aligned}\Delta\text{KE} &= qV_{\text{AB}} = qEd \\ &= (1.60\times 10^{-19}\text{ C})(2.00\times 10^{6}\text{ V/m})(0.400\,\text{m})\left(\frac{1\,\text{eV}}{1.60\times 10^{-19}\text{ J}}\right)\left(\frac{1\,\text{keV}}{1000\,\text{eV}}\right) \\ &= \underline{800\,\text{keV}}\end{aligned}$$

In other words, the electron would gain 800 keV of energy if accelerated over a distance of 0.400 m.

(b) Using the same expression in part (a), we can now solve for the distance traveled:

$$d = \frac{\Delta \text{KE}}{qE} = \frac{(50.0\times 10^{9}\text{ eV})}{(1.60\times 10^{-19}\text{ C})(2.00\times 10^{6}\text{ v/m})}\left(\frac{1.60\times 10^{-19}\text{ J}}{1\text{ eV}}\right)$$
$$= 2.50\times 10^{4}\text{ m} = \underline{25.0\text{ km}}$$

So, the electron must be accelerated over a distance of 25.0 km to gain 50.0 GeV of energy.

19.3 ELECTRIC POTENTIAL DUE TO A POINT CHARGE

29. *If the potential due to a point charge is* $5.00\times 10^{2}\text{ V}$ *at a distance of 15.0 m, what are the sign and magnitude of the charge?*

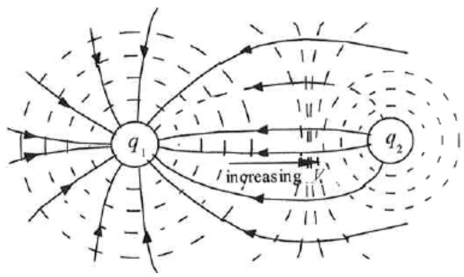
Solution Given the equation $V = \dfrac{kQ}{r}$, we can determine the charge given the potential and the separation distance: $Q = \dfrac{rV}{k} = \dfrac{(15.0\text{ m})(500\text{ V})}{9.00\times 10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2} = \underline{8.33\times 10^{-7}\text{ C}}$

The charge is positive because the potential is positive.

19.4 EQUIPOTENTIAL LINES

38. *Figure 19.28 shows the electric field lines near two charges* q_1 *and* q_2*, the first having a magnitude four times that of the second. Sketch the equipotential lines for these two charges, and indicate the direction of increasing potential.*

Solution To draw the equipotential lines, remember that they are always perpendicular to electric fields lines. The potential is greatest (most positive) near the positive charge, $q_{2,}$ and least (most negative) near the negative charge, $q_{1.}$ In other words, the potential increases as you move out from the charge q_1, and it increases as you move towards the charge q_2.



19.5 CAPACITORS AND DIELETRICS

46. *What charge is stored in a* $180\ \mu\text{F}$ *capacitor when 120 V is applied to it?*

Solution Using the equation $Q = CV$, we can determine the charge on a capacitor, since we are given its capacitance and its voltage:

$$Q = CV = (1.80\times10^{-4}\ \text{F})(120\ \text{V}) = 2.16\times10^{-2} = \underline{21.6\ \text{mC}}$$

50. *What voltage must be applied to an 8.00 nF capacitor to store 0.160 mC of charge?*

Solution Using the equation $Q = CV$, we can determine the voltage that must be applied to a capacitor, given the charge it stores and its capacitance:

$$V = \frac{Q}{C} = \frac{1.60\times10^{-4}\ \text{C}}{8.00\times10^{-9}\ \text{F}} = 2.00\times10^{4}\ \text{V} = \underline{20.0\ \text{kV}}$$

19.6 CAPACITORS IN SERIES AND PARALLEL

59. *What total capacitances can you make by connecting a* $5.00\ \mu\text{F}$ *and an* $8.00\ \mu\text{F}$ *capacitor together?*

Solution There are two ways in which you can connect two capacitors: in parallel and in series. When connected in series, the total capacitance is given by the equation

$$\frac{1}{C_s}=\frac{1}{C_1}+\frac{1}{C_2}\Rightarrow C_s=\frac{C_1C_2}{C_1+C_2}=\frac{(5.00\ \mu\text{F})(8.00\ \mu\text{F})}{5.00\ \mu\text{F}+8.00\ \mu\text{F}}=\underline{3.08\ \mu\text{F (series)}}$$

and when connected in parallel, the total capacitance is given by the equation

$$C_p=C_1+C_2=5.00\ \mu\text{F}+8.00\ \mu\text{F}=\underline{13.0\ \mu\text{F(parallel)}}$$

19.7 ENERGY STORED IN CAPACITORS

66. *Suppose you have a 9.00 V battery, a* $2.00\ \mu\text{F}$ *capacitor, and a* $7.40\ \mu\text{F}$ *capacitor. (a) Find the charge and energy stored if the capacitors are connected to the battery in series. (b) Do the same for a parallel connection.*

Solution (a) If the capacitors are connected in series, their total capacitance is:

$$\frac{1}{C_s}=\frac{1}{C_1}+\frac{1}{C_2}\Rightarrow C_s=\frac{C_1C_2}{C_1+C_2}=\frac{(2.00\ \mu\text{F})(7.40\ \mu\text{F})}{9.40\ \mu\text{F}}=1.575\ \mu\text{F}.$$

Then, since we know the capacitance and the voltage of the battery, we can use the equation $Q=CV$ to determine the charge stored in the capacitors:

$$Q=C_sV=(1.574\times10^{-6}\ \text{F})(9.00\ \text{V})=\underline{1.42\times10^{-5}\ \text{C}}$$

Then determine the energy stored in the capacitors, using the equation

$$E_{cap}=\frac{C_sV^2}{2}=\frac{(1.574\times10^{-6}\ \text{F})(9.00\ \text{V})^2}{2}=\underline{6.38\times10^{-5}\ \text{J}}.$$

Note: by using the form of this equation $E=\dfrac{CV^2}{2}$ involving capacitance and voltage, we can avoid using one of the parameters that we calculated, minimizing our change of propagating an error.

(b) If the capacitors are connected in parallel, their total capacitance is given by the equation $C_\text{p} = C_1 + C_2 = 2.00\ \mu\text{F} + 7.40\ \mu\text{F} = 9.40\ \mu\text{F}$

Again, we use the equation $Q = CV$ to determine the charge stored in the capacitors: $Q = C_\text{p}V = (9.40\times10^{-6}\ \text{F})(9.00\ \text{V}) = \underline{8.46\times10^{-5}\ \text{C}}$

And finally, using the following equation again, we can determine the energy stored in the capacitors:

$$E_\text{cap} = \frac{C_\text{p}V^2}{2} = \frac{(9.40\times10^{-6}\ \text{F})(9.00\ \text{V})^2}{2} = \underline{3.81\times10^{-4}\ \text{J}}$$

CHAPTER 20: ELECTRIC CURRENT, RESISTANCE, AND OHM'S LAW

20.1 CURRENT

1. *What is the current in milliamperes produced by the solar cells of a pocket calculator through which 4.00 C of charge passes in 4.00 h?*

Solution Using the equation $I = \frac{\Delta Q}{\Delta t}$, we can calculate the current given in the charge and the time, remembering that $1\ \text{A} = 1\ \text{C/s}$:

$$I = \frac{\Delta Q}{\Delta t} = \frac{4.00\ \text{C}}{4.00\ \text{h}}\left(\frac{1\ \text{h}}{3600\ \text{s}}\right) = 2.778\times10^{-4}\ \text{A} = \underline{0.278\ \text{mA}}$$

7. *(a) A defibrillator sends a 6.00-A current through the chest of a patient by applying a 10,000-V potential as in Figure 20.38. What is the resistance of the path? (b) The defibrillator paddles make contact with the patient through a conducting gel that greatly reduces the path resistance. Discuss the difficulties that would ensue if a larger voltage were used to produce the same current through the patient, but with the path having perhaps 50 times the resistance. (Hint: The current must be about the same, so a higher voltage would imply greater power. Use this equation for power: $P = I^2R$.)*

Solution (a) Using the equation $I = \frac{V}{R}$, we can calculate the resistance of the path given the current and the potential: $I = \frac{V}{R}$, so that

$$R = \frac{V}{I} = \frac{10{,}000\ \text{V}}{6.00\ \text{A}} = 1.667\times10^{3}\ \Omega = \underline{1.67\ \text{k}\Omega}$$

(b) If a 50 times larger resistance existed, keeping the current about the same, the power would be increased by a factor of about 50, causing much more energy to be transferred to the skin, which could cause serious burns. The gel used reduces the resistance, and therefore reduces the power transferred to the skin.

13. *A large cyclotron directs a beam of* He^{++} *nuclei onto a target with a beam current of 0.250 mA. (a) How many* He^{++} *nuclei per second is this? (b) How long does it take for 1.00 C to strike the target? (c) How long before 1.00 mol of* He^{++} *nuclei strike the target?*

Solution (a) Since we know that a He^{++} ion has a charge of twice the basic unit of charge, we can convert the current, which has units of C/s, into the number of He^{++} ions per second: $(2.50\times10^{-4}\text{ C/s})\dfrac{1\text{ He}^{++}}{2(1.60\times10^{-19}\text{ C})}=\underline{7.81\times10^{14}\text{ He}^{++}\text{nuclei/s}}$

(b) Using the equation $I=\dfrac{\Delta Q}{\Delta t}$, we can determine the time it takes to transfer 1.00 C of charge, since we know the current: $I=\dfrac{\Delta Q}{\Delta t}$, so that

$$\Delta t=\frac{\Delta Q}{I}=\frac{1.00\text{ C}}{2.50\times10^{-4}\text{ A}}=\underline{4.00\times10^{3}\text{ s}}$$

(c) Using the result from part (a), we can determine the time it takes to transfer 1.00 mol of He^{++} ions by converting units:

$$(1.00\text{ mol He}^{++})\left(\frac{6.02\times10^{23}\text{ ions}}{\text{mol}}\right)\left(\frac{1\text{ s}}{7.81\times10^{14}\text{ He}^{++}\text{ions}}\right)=\underline{7.71\times10^{8}\text{ s}}$$

20.2 OHM'S LAW: RESISTANCE AND SIMPLE CIRCUITS

19. *Calculate the effective resistance of a pocket calculator that has a 1.35-V battery and through which 0.200 mA flows.*

Solution Using the equation $I = \frac{V}{R}$, given the voltage and the current, we can determine the resistance: $I = \frac{V}{R}$, so that $R = \frac{V}{I} = \frac{1.35\text{ V}}{2.00\times10^{-4}\text{ A}} = 6.75\times10^{3}\,\Omega = \underline{6.75\text{ k}\Omega}$

20.3 RESISTANCE AND RESISTIVITY

25. *The diameter of 0-gauge copper wire is 8.252 mm. Find the resistance of a 1.00-km length of such wire used for power transmission.*

Solution We know we want to use the equation $R = \frac{\rho L}{A}$, so we need to determine the radius for the cross-sectional area of $A = \pi r^2$. Since we know the diameter of the wire is 8.252 mm, we can determine the radius of the wire:

$$r = \frac{d}{2} = \frac{8.252\times10^{-3}\text{ m}}{2} = 4.126\times10^{-3}\text{ m}.$$

We also know from Table 20.1 that the resistivity of copper is $1.72\times10^{-8}\ \Omega\text{m}$. These values give a resistance of: $R = \frac{\rho L}{A} = \frac{(1.72\times10^{-8}\ \Omega\cdot\text{m})(1.00\times10^{3}\text{ m})}{\pi(4.126\times10^{-3}\text{ m})^2} = \underline{0.322\,\Omega}$

31. *Of what material is a resistor made if its resistance is 40.0% greater at* $100°\text{C}$ *than at* $20.0°\text{C}$*?*

Solution We can use the equation $R = R_0(1+\alpha\Delta T)$ to determine the temperature coefficient of resistivity of the material. Then, by examining Table 20.2, we can determine the type of material used to make the resistor. Since $R = R_0(1+\alpha\Delta T) = 1.400R_0$, for a temperature change of $80.0°\text{C}$, we can determine α:

$$\alpha\Delta T = 1.400 - 1 \Rightarrow \alpha = \frac{0.400}{\Delta T} = \frac{0.400}{80.0°\text{ C}} = \underline{5.00\times10^{-3}\ /°\text{C}}$$

So, based on the values of in Table 20.2, the resistor is made of iron.

37. *(a) Digital medical thermometers determine temperature by measuring the resistance of a semiconductor device called a thermistor (which has $\alpha = -0.0600/°\text{C}$) when it is at the same temperature as the patient. What is a patient's temperature if the thermistor's resistance at that temperature is 82.0% of its value at $37.0°\text{C}$ (normal body temperature)? (b) The negative value for α may not be maintained for very low temperatures. Discuss why and whether this is the case here. (Hint: Resistance can't become negative.)*

Solution (a) $R = R_0\left[1+\alpha\,(T-37.0°\text{ C})\right] = 0.820\,R_0,\text{ where }\alpha = -0.600/°\text{C}.$

Dividing by $R_0, 1+\alpha(T-37.0°\text{ C}) = 0.820$, so that $0.180 = -\alpha\,(T-37.0°\text{ C})$,

giving $(T-37.0°\text{ C}) = \dfrac{-0.180}{\alpha} = \dfrac{-0.180}{-0.0600/°\text{C}} = 3.00°\text{ C}.$

Finally, $T = 37.0°\text{ C} + 3.00°\text{ C} = \underline{40.0°\text{ C}}$

(b) If α is negative at low temperatures, then the term $\left[1+\alpha(T-37.0°\text{ C})\right]$ can become negative, which implies that the resistance has the opposite sign of the initial resistance , or it has become negative. Since it is not possible to have a negative resistance, the temperature coefficient of resistivity cannot remain negative to low temperatures. In this example the magnitude is $\underline{\alpha \rangle \dfrac{1}{37.0°\text{ C}-T}}$

39. ***Unreasonable Results*** *(a) To what temperature must you raise a resistor made of constantan to double its resistance, assuming a constant temperature coefficient of resistivity? (b) To cut it in half? (c) What is unreasonable about these results? (d) Which assumptions are unreasonable, or which premises are inconsistent?*

Solution (a) Using the equation $R = R_0(1+\alpha\Delta T)$ and setting the resistance equal to twice the initial resistance, we can solve for the final temperature:

$$R = R_0(1+\alpha\Delta T) = 2R_0 \Rightarrow \alpha\Delta T = \alpha(T-T_0) = 1.\ \left(T_0 = 20°\text{C}\right)$$

So the final temperature will be:

$$T - T_0 = \frac{1}{\alpha} = \frac{1}{2\times10^{-6}\,/\,^\circ\text{C}} = 5\times10^5\ ^\circ\text{C} \Rightarrow T = \underline{5\times10^5\ ^\circ\text{C}}$$

(b) Again, using the equation $R = R_0(1+\alpha\Delta T)$, we can solve for the final temperature when the resistance is half the initial resistance:

$R = R_0(1+\alpha\Delta T) = \frac{R_0}{2} \Rightarrow \alpha(T - T_0) = -\frac{1}{2}$, so the final temperature will be:

$$T - T_0 = \frac{-0.5}{2\times10^{-6}\,/\,^\circ\text{C}} = -2.5\times10^5\ ^\circ\text{C or } T = \underline{-2.5\times10^5\ ^\circ\text{C}}.$$

(c) In part (a), the temperature is above the melting point of any metal. In part (b) the temperature is far below 0 K, which is impossible.

(d) The assumption that the resistivity for constantan will remain constant over the derived temperature ranges in part (a) and (b) above is wrong. For large temperature changes, α may vary, or a non-linear equation may be needed to find ρ.

20.4 ELECTRIC POWER AND ENERGY

45. *Verify that the units of a volt-ampere are watts, as implied by the equation $P = IV$.*

Solution Starting with the equation $P = IV$, we can get an expression for a watt in terms of current and voltage: $[P] = \text{W}$, $[IV] = \text{A.V} = (\text{C/s})(\text{J/C}) = \text{J/s} = \text{W}$, so that a watt is equal to an ampere - volt.

55. *A cauterizer, used to stop bleeding in surgery, puts out 2.00 mA at 15.0 kV. (a) What is its power output? (b) What is the resistance of the path?*

Solution (a) Using the equation $P = IV$, we can determine the rms power given the current and the voltage:

$$P = IV = (2.00\times10^{-3}\text{ A})(15.0\times10^3\text{ V}) = \underline{30.0\text{ W}}$$

(b) Now, using the equation $I = \frac{V}{R}$, we can solve for the resistance, without using the result from part (a): $I = \frac{V}{R} \Rightarrow R = \frac{V}{I} = \frac{(1.50 \times 10^4 \text{ V})}{2.00 \times 10^{-3} \text{ A}} = 7.50 \times 10^6 \ \Omega = \underline{7.50 \text{ M}\Omega}$

Note, this assume the cauterizer obeys Ohm's law, which will be true for ohmic materials like good conductors.

59. ***Integrated Concepts*** *Cold vaporizers pass a current through water, evaporating it with only a small increase in temperature. One such home device is rated at 3.50 A and utilizes 120 V AC with 95.0% efficiency. (a) What is the vaporization rate in grams per minute? (b) How much water must you put into the vaporizer for 8.00 h of overnight operation? (See Figure 20.42.)*

Solution

(a) From the equation $P = IV$, we can determine the power generated by the vaporizer. $P = IV = (3.50 \text{ A})(120 \text{ V}) = 420 \text{ J/s} = 0.420 \text{ kJ/s}$ and since the vaporizer has an efficiency of 95.0%, the heat that is capable of vaporizing the water is $Q = (0.950)\ Pt$. This heat vaporizes the water according to the equation $Q = mL_\text{v}$, where $L_\text{v} = 2256 \text{ kJ/kg}$, from Table 14.2, so that $(0.950)\ Pt = mL_\text{v}$, or

$$m = \frac{(0.950)Pt}{L_\text{v}} = \frac{(0.950)(0.420 \text{ kJ/s})(60.0 \text{ s})}{2256 \text{ kJ/kg}} = 0.0106 \text{ kg} \Rightarrow \underline{10.6 \text{ g/min}}$$

(b) If the vaporizer is to run for 8.00 hours , we need to calculate the mass of the water by converting units:

$$m_\text{required} = (10.6 \text{ g/min})(8.0 \text{ h})\left(\frac{60 \text{ min}}{1 \text{ h}}\right) = 5.09 \times 10^3 \text{ g} = \underline{5.09 \text{ kg}}$$

In other words, making use of Table 11.1 to get the density of water, it requires $5.09 \text{ kg} \times \frac{\text{m}^3}{10^3 \text{ kg}} \times \frac{\text{L}}{10^{-3} \text{ m}^3} = \underline{5.09 \text{ L}}$ of water to run overnight.

65. ***Integrated Concepts*** *A light-rail commuter train draws 630 A of 650-V DC electricity when accelerating. (a) What is its power consumption rate in kilowatts? (b) How long does it take to reach 20.0 m/s starting from rest if its loaded mass is* $5.30\times10^4\ \text{kg}$*, assuming 95.0% efficiency and constant power? (c) Find its average acceleration. (d) Discuss how the acceleration you found for the light-rail train compares to what might be typical for an automobile.*

Solution

(a) Using the equation $P = IV$, we can determine the power generated:

$$P = IV = (630\ \text{A})(650\ \text{V}) = 4.10\times10^5\ \text{W} = 410\ \text{kW}$$

(b) Since the efficiency is 95.0%, the effective power is $P_{\text{effective}} = (0.950)P = 389.0\ \text{kW}$. Then we can calculate the work done by the train: $W = (P_{\text{eff}})t$. Setting that equal to the change in kinetic energy gives us an expression for the time it takes to reach 20.0 m/s from rest:

$W = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^{\ 2} = (P_{\text{eff}})t$, so that

$$t = \frac{(1/2)mv^2 - (1/2)mv_0^{\ 2}}{P_{\text{eff}}} = \frac{0.5(5.30\times10^4\ \text{kg})(20.0\ \text{m/s})^2}{3.890\times10^5\ \text{W}} = 27.25\ \text{s} = \underline{27.3\ \text{s}}$$

(c) We recall that $v = v_0 + at$, so that $a = \frac{v}{t} = \frac{20.0\ \text{m/s}}{27.25\ \text{s}} = \underline{0.734\ \text{m/s}^2}$

(d) A typical automobile can go from 0 to 60 mph in 10 seconds, so that its acceleration is: $a = \frac{v}{t} = \frac{60\ \text{mi/hr}}{10\ \text{s}}\times\frac{1\ \text{hr}}{3600\ \text{s}}\times\frac{1609\ \text{m}}{\text{mi}} = 2.7\ \text{m/s}^2$

Thus, a light-rail train accelerates much slower than a car, but it can reach final speeds substantially faster than a car can sustain. So, typically light-rail tracks are very long and straight, to allow them to reach these faster final speeds without decelerating around sharp turns.

20.5 ALTERNATING CURRENT VERSUS DIRECT CURRENT

73. *Certain heavy industrial equipment uses AC power that has a peak voltage of 679 V. What is the rms voltage?*

Solution Using the equation $V_{\text{rms}} = \dfrac{V_0}{\sqrt{2}}$, we can determine the rms voltage, given the peak voltage: $V_{\text{rms}} = \dfrac{V_0}{\sqrt{2}} = \dfrac{679\ \text{V}}{\sqrt{2}} = \underline{480\ \text{V}}$

79. *What is the peak power consumption of a 120-V AC microwave oven that draws 10.0 A?*

Solution Using the equation $P_{\text{ave}} = I_{\text{rms}}V_{\text{rms}}$, we can calculate the average power given the rms values for the current and voltage: $P_{\text{ave}} = I_{\text{rms}}V_{\text{rms}} = (10.0\ \text{A})(120\ \text{V}) = 1.20\ \text{kW}$

Next, since the peak power is the peak current times the peak voltage:

$$P_0 = I_0V_0 = 2(\frac{1}{2}I_0V_0) = 2P_{\text{ave}} = \underline{2.40\ \text{kW}}$$

83. *Find the time after $t = 0$ when the instantaneous voltage of 60-Hz AC first reaches the following values: (a) $V_0/2$ (b) V_0 (c) 0.*

Solution (a) From the equation $V = V_0 \sin 2\pi ft$, we know how the voltage changes with time for an alternating current (AC). So, if we want the voltage to be equal to $\dfrac{V_0}{2}$, we know that $\dfrac{V_0}{2} = V_0 \sin 2\pi ft$, so that: $\sin 2\pi ft = \dfrac{1}{2}$, or $t = \dfrac{\sin^{-1}(0.5)}{2\pi f}$. Since we have a frequency of 60 Hz, we can solve for the time that this first occurs (remembering to have your calculator in radians):

$$t=\frac{0.5236\text{ rad}}{2\pi\ (60\text{ Hz})}=1.39\times10^{-3}\text{ s}=\underline{1.39\text{ ms}}$$

(b) Similarly, for $V=V_0: V=V_0\sin 2\pi ft=V_0$, so that

$$t=\frac{\sin^{-1}1}{2\pi f}=\frac{\pi/2\text{ rad}}{2\pi(60\text{ Hz})}=4.17\times10^{-3}\text{ s}=\underline{4.17\text{ ms}}$$

(c) Finally, for $V=0$: $V=V_0\sin 2\pi ft=0$, so that $2\pi ft=0,\pi,2\pi,\ldots$, or for the first time after $t=0$: $2\pi ft=\pi$, or $t=\dfrac{1}{2(60\text{ Hz})}=8.33\times10^{-3}\text{s}=\underline{8.33\text{ ms}}$

20.6 ELECTRIC HAZARDS AND THE HUMAN BODY

89. *Foolishly trying to fish a burning piece of bread from a toaster with a metal butter knife, a man comes into contact with 120-V AC. He does not even feel it since, luckily, he is wearing rubber-soled shoes. What is the minimum resistance of the path the current follows through the person?*

Solution

From Table 20.3, we know that the threshold of sensation is $I=1.00\text{ mA}$. The minimum resistance for the shock to not be felt will occur when I is equal to this value. So, using the equation $I=\dfrac{V}{R}$,we can determine the minimum resistance for 120 V AC current: $R=\dfrac{V}{I}=\dfrac{120\text{ V}}{1.00\times10^{-3}\text{ A}}=\underline{1.20\times10^{5}\ \Omega}$

CHAPTER 21: CIRCUITS, BIOELECTRICITY, AND DC INSTRUMENTS

21.1 RESISTORS IN SERIES AND PARALLEL

1. *(a) What is the resistance of ten* $275\text{-}\Omega$ *resistors connected in series? (b) In parallel?*

Solution (a) From the equation $R_s = R_1 + R_2 + R_3 + \ldots$ we know that resistors in series add:

$$R_s = R_1 + R_2 + R_3 + \ldots\ldots + R_{10} = (275\ \Omega)(10) = \underline{2.75\ \text{k}\Omega}$$

(b) From the equation $\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots$, we know that resistors in series add like:

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots\ldots + \frac{1}{R_{10}} = (10)\left(\frac{1}{275\ \Omega}\right) = 3.64\times10^{-2}/\Omega$$

So that $R_p = \left(\frac{1}{3.64\times10^{-2}}\right)\Omega = \underline{27.5\ \Omega}$

7. *Referring to the example combining series and parallel circuits and Figure 21.6, calculate* I_3 *in the following two different ways: (a) from the known values of* I *and* I_2*; (b) using Ohm's law for* R_3*. In both parts explicitly show how you follow the steps in the Problem-Solving Strategies for Series and Parallel Resistors.*

Solution ***Step 1:*** The circuit diagram is drawn in Figure 21.6.

Step 2: Find I_3.

Step 3: Resistors R_2 and R_3 are in parallel. Then, resistor R_1 is in series with the combination of R_2 and R_3.

Step 4:

(a) Looking at the point where the wire comes into the parallel combination of R_2 and R_3, we see that the current coming in I is equal to the current going out I_2 and I_3, so that $I = I_2 + I_3$, or $I_3 = -I_2 = 2.35\text{ A} - 1.61\text{ A} = \underline{0.74\text{ A}}$

(b) Using Ohm's law for R_3, and voltage for the combination of R_2 and R_3, found in Example 21.3, we can determine the current: $I_3 = \dfrac{V_\text{p}}{R_3} = \dfrac{9.65\text{ V}}{13.0\ \Omega} = \underline{0.742\text{ A}}$

Step 5: The result is reasonable because it is smaller than the incoming current, I, and both methods produce the same answer.

21.2 ELECTROMOTIVE FORCE: TERMINAL VOLTAGE

15. *Carbon-zinc dry cells (sometimes referred to as non-alkaline cells) have an emf of 1.54 V, and they are produced as single cells or in various combinations to form other voltages. (a) How many 1.54-V cells are needed to make the common 9-V battery used in many small electronic devices? (b) What is the actual emf of the approximately 9-V battery? (c) Discuss how internal resistance in the series connection of cells will affect the terminal voltage of this approximately 9-V battery.*

Solution (a) To determine the number simply divide the 9-V by the emf of each cell:

$$9\text{ V} \div 1.54\text{ V} = 5.84 \Rightarrow \underline{6}$$

(b) If six dry cells are put in series , the actual emf is $1.54\text{ V} \times 6 = \underline{9.24\text{ V}}$

(c) Internal resistance will decrease the terminal voltage because there will be voltage drops across the internal resistance that will not be useful in the operation of the 9-V battery.

30. ***Unreasonable Results*** *(a) What is the internal resistance of a 1.54-V dry cell that supplies 1.00 W of power to a* $15.0\text{-}\Omega$ *bulb? (b) What is unreasonable about this result? (c) Which assumptions are unreasonable or inconsistent?*

Solution (a) Using the equation $P = I^2 R$, we have $I = \sqrt{\frac{P}{R}} = \sqrt{\frac{1.00\text{ W}}{15.0\ \Omega}} = 0.258\text{ A}$. So using Ohm's Law and $V = E - Ir$ we have

$$V = E - Ir = IR\text{, or}: r = \frac{E}{I} - R = \frac{1.54\text{ V}}{0.258\text{ A}} - 15.0\ \Omega = \underline{-9.04\ \Omega}$$

(b) You cannot have negative resistance.

(c) The voltage should be less than the emf of the battery; otherwise the internal resistance comes out negative. Therefore, the power delivered is too large for the given resistance, leading to a current that is too large.

21.3 KIRCHHOFF'S RULES

31. *Apply the loop rule to loop abcdefgha in Figure 21.25.*

Solution Using the loop rule for loop abcdefgha in Figure 21.25 gives:

$$\underline{-I_2R_3 + E_1 - I_2r_1 + I_3r_3 + I_3r_2 - E_2 = 0}$$

37. *Apply the loop rule to loop akledcba in Figure 21.52.*

Solution Using the loop rule to loop akledcba in Figure 21.52 gives:

$$\underline{E_2 - I_2r_2 - I_2R_2 + I_1R_5 + I_1r_1 - E_1 + I_1R_1 = 0}$$

21.4 DC VOLTMETERS AND AMMETERS

44. *Find the resistance that must be placed in series with a $25.0\text{-}\Omega$ galvanometer having a $50.0\text{-}\mu\text{A}$ sensitivity (the same as the one discussed in the text) to allow it to be used as a voltmeter with a 0.100-V full-scale reading.*

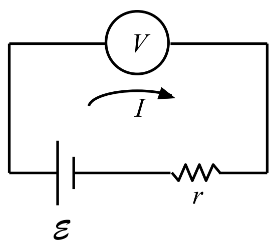
Solution We are given $r = 25.0\,\Omega$, $V = 0.200\,\text{V}$, and $I = 50.0\,\mu\text{A}$.

Since the resistors are in series, the total resistance for the voltmeter is found by using $R_s = R_1 + R_2 + R_3 + \ldots$. So, using Ohm's law we can find the resistance R:

$$R_{\text{tot}} = R + r = \frac{V}{I}, \text{so that } R = \frac{V}{I} - r = \frac{0.100\ \text{V}}{5.00\times 10^{-5}\ \text{A}} - 25.0\ \Omega = 1975\ \Omega = \underline{1.98\ \text{k}\Omega}$$

50. *Suppose you measure the terminal voltage of a 1.585-V alkaline cell having an internal resistance of $0.100\,\Omega$ by placing a $1.00\text{-k}\,\Omega$ voltmeter across its terminals. (Figure 21.54.) (a) What current flows? (b) Find the terminal voltage. (c) To see how close the measured terminal voltage is to the emf, calculate their ratio.*

Solution (a)



Going counterclockwise around the loop using the loop rule gives:

$$-E + Ir + IR = 0, \text{ or}$$

$$I = \frac{E}{R + r} = \frac{1.585\,\text{V}}{\left(1.00\times 10^{3}\ \Omega\right) + 0.100\,\Omega} = 1.5848\times 10^{3}\ \text{A} = \underline{1.58\times 10^{3}\ \text{A}}$$

(b) The terminal voltage is given by the equation $V = E - Ir$:

$$V = E - Ir = 1.585\,\text{V} - \left(1.5848\times 10^{3}\ \text{A}\right)\left(0.100\,\Omega\right) = \underline{1.5848\,\text{V}}$$

Note: The answer is reported to 5 significant figures to see the difference.

(c) To calculate the ratio, divide the terminal voltage by the emf:

$$\frac{V}{E} = \frac{1.5848\,\text{V}}{1.585\,\text{V}} = \underline{0.99990}$$

21.5 NULL MEASUREMENTS

58. *Calculate the* emf_x *of a dry cell for which a potentiometer is balanced when* $R_\text{x} = 1.200\ \Omega$*, while an alkaline standard cell with an emf of 1.600 V requires* $R_\text{s} = 1.247\ \Omega$ *to balance the potentiometer.*

Solution We know $E_\text{x} = IR_\text{x}$ and $E_\text{s} = IR_\text{s}$, so that

$$\frac{E_\text{x}}{E_\text{s}} = \frac{I_\text{x}}{I_\text{s}} = \frac{R_\text{x}}{R_\text{s}}, \text{ or } E_\text{x} = E_\text{s}\left(\frac{R_\text{x}}{R_\text{s}}\right) = (1.600\ \text{V})\left(\frac{1.200\ \Omega}{1.247\ \Omega}\right) = \underline{1.540\ \text{V}}$$

21.6 DC CIRCUITS CONTAINING RESISTORS AND CAPACITORS

63. *The timing device in an automobile's intermittent wiper system is based on an* RC *time constant and utilizes a* $0.500\text{-}\mu\text{F}$ *capacitor and a variable resistor. Over what range must* R *be made to vary to achieve time constants from 2.00 to 15.0 s?*

Solution From the equation $\tau = RC$, we know that:

$$R = \frac{\tau}{C} = \frac{2.00\ \text{s}}{5.00\times 10^{-7}\ \text{F}} = 4.00\times 10^{6}\ \Omega \text{ and } R = \frac{\tau}{C} = \frac{15.0\ \text{s}}{5.00\times 10^{-7}\ \text{F}} = 3.00\times 10^{7}\ \Omega$$

Therefore, the range for R is: $\underline{4.00\times 10^{6}\ \Omega - 3.00\times 10^{7}\ \Omega} = \underline{4.00 \text{ to } 30.0\ \text{M}\Omega}$

69. *A heart defibrillator being used on a patient has an* RC *time constant of 10.0 ms due to the resistance of the patient and the capacitance of the defibrillator. (a) If the defibrillator has an* $8.00\text{-}\mu\text{F}$ *capacitance, what is the resistance of the path through the patient? (You may neglect the capacitance of the patient and the resistance of the defibrillator.) (b) If the initial voltage is 12.0 kV, how long does it take to decline to* $6.00\times 10^{2}\ \text{V}$*?*

Solution (a) Using the equation $\tau = RC$ we can calculate the resistance:

$$R = \frac{\tau}{C} = \frac{1.00\times10^{-2}\ \text{s}}{8.00\times10^{-6}\ \text{F}} = 1.25\times10^{3}\ \Omega = \underline{1.25\ \text{k}\Omega}$$

(b) Using the equation $V = V_0 e^{-\tau/RC}$, we can calculate the time it takes for the voltage to drop from $12.0\ \text{kV to } 600\ \text{V}$:

$$\tau = -RC\ \ln\left(\frac{V}{V_0}\right)$$

$$= -\left(1.25\times10^{3}\ \Omega\right)\left(8.00\times10^{-6}\ \text{F}\right)\ln\left(\frac{600\ \text{V}}{1.20\times10^{4}\ \text{V}}\right) = 2.996\times10^{-2}\ \text{s} = \underline{30.0\ \text{ms}}$$

74. ***Integrated Concepts*** *If you wish to take a picture of a bullet traveling at 500 m/s, then a very brief flash of light produced by an RC discharge through a flash tube can limit blurring. Assuming 1.00 mm of motion during one RC constant is acceptable, and given that the flash is driven by a $600\text{-}\mu\text{F}$ capacitor, what is the resistance in the flash tube?*

Solution Using $c = \frac{x}{t}$ or $t = \frac{x}{v}$ and the equation for the time constant, we can write the time constant as $\tau = RC$, so getting these two times equal gives an expression from which we can solve for the required resistance:

$$\frac{x}{v} = RC, \text{or } R = \frac{x}{vC} = \frac{1.00\times10^{-3}\ \text{m}}{(500\ \text{m/s})(6.00\times10^{-4}\ \text{F})} = \underline{3.33\times10^{-3}\ \Omega}$$

CHAPTER 22: MAGNETISM

22.4 MAGNETIC FIELD STRENGTH: FORCE ON A MOVING CHARGE IN A MAGNETIC FIELD

1. *What is the direction of the magnetic force on a positive charge that moves as shown in each of the six cases shown in Figure 22.50?*

Solution Use the right hand rule-1 to solve this problem. Your right thumb is in the direction of velocity, your fingers point in the direction of magnetic field, and then your palm points in the direction of magnetic force.

(a) Your right thumb should be facing down, your fingers out of the page, and then the palm of your hand points to the left (West).

(b) Your right thumb should point up, your fingers should point to the right, and then the palm of your hand points into the page.

(c) Your right thumb should point to the right, your fingers should point into the page, and then the palm of your hand points up (North).

(d) The velocity and the magnetic field are anti-parallel, so there is no force.

(e) Your right thumb should point into the page, your fingers should point up, and then the palm of your hand points to the right (East).

(f) Your right thumb should point out of the page, your fingers should point to the left, and then the palm of your hand points down (South).

7. *What is the maximum force on an aluminum rod with a* $0.100\text{-}\mu\text{C}$ *charge that you pass between the poles of a 1.50-T permanent magnet at a speed of 5.00 m/s? In what direction is the force?*

Solution Examining the equation $F = qvB\sin\theta$, we see that the maximum force occurs when $\sin\theta = 1$, so that: $F_{\max} = qvB = (0.100\times10^{-6}\text{ C})(5.00\text{ m/s})(1.50\text{ T}) = \underline{7.50\times10^{-7}\text{ N}}$

22.5 FORCE ON A MOVING CHARGE IN A MAGNETIC FIELD: EXAMPLES AND APPLICATIONS

13. *A proton moves at* $7.50\times10^{7}\ \text{m/s}$ *perpendicular to a magnetic field. The field causes the proton to travel in a circular path of radius 0.800 m. What is the field strength?*

Solution Using the equation $r=\dfrac{mv}{qB}$, we can solve for the magnetic field strength necessary to move the proton in a circle of radius 0.800 m:

$$B=\frac{mv}{qr}=\frac{(1.67\times10^{-27}\ \text{kg})(7.50\times10^{7}\ \text{m/s})}{(1.60\times10^{-19}\ \text{C})(0.800\ \text{m})}=\underline{0.979\ \text{T}}$$

19. *(a) At what speed will a proton move in a circular path of the same radius as the electron in Exercise 22.12? (b) What would the radius of the path be if the proton had the same speed as the electron? (c) What would the radius be if the proton had the same kinetic energy as the electron? (d) The same momentum?*

Solution (a) Since we know $r=\dfrac{mv}{qB}$, and we want the radius of the proton to equal the radius of the electron in Exercise 22.12, we can write the velocity of the proton in terms of the information we know about the electron:

$$v_{\text{p}}=\frac{q_{\text{p}}Br}{m_{\text{p}}}=\frac{q_{\text{p}}B}{m_{\text{p}}}\left(\frac{m_{\text{e}}v_{\text{e}}}{q_{\text{e}}B}\right)=\frac{m_{\text{e}}v_{\text{e}}}{m_{\text{p}}}$$

$$=\frac{(9.11\times10^{-31}\ \text{kg})\ (7.50\times10^{6}\ \text{m/s})}{1.67\times10^{-27}\ \text{kg}}=\underline{4.09\times10^{3}\ \text{m/s}}$$

(b) Now, using $r=\dfrac{mv}{qB}$, we can solve for the radius of the proton if the velocity equals the velocity of the electron:

$$r_{\text{p}}=\frac{mv_{\text{e}}}{qB}=\frac{(1.67\times10^{-27}\ \text{kg})(7.50\times10^{6}\ \text{m/s})}{(1.60\times10^{-19}\ \text{C})(1.00\times10^{-5}\ \text{T})}=\underline{7.83\times10^{3}\ \text{m}}$$

(c) First, we need to determine the speed of the proton if the kinetic energies were the same: $\dfrac{1}{2}m_{\text{e}}v_{\text{e}}^{2}=\dfrac{1}{2}m_{\text{p}}v_{\text{p}}^{2}$, so that

$$v_p = v_e\sqrt{\frac{m_e}{m_p}} = (7.50\times10^6\ \text{m/s})\sqrt{\frac{9.11\times10^{-31}\ \text{kg}}{1.67\times10^{-27}\ \text{kg}}} = 1.75\times10^5\ \text{m/s}$$

Then using $r = \frac{mv}{qB}$, we can determine the radius:

$$r = \frac{mv}{qB} = \frac{(1.67\times10^{-27}\ \text{kg})\ (1.752\times10^5\ \text{m/s})}{(1.60\times10^{-19}\ \text{C})\ (1.00\times10^{-5}\ \text{T})} = \underline{1.83\times10^2\ \text{m}}$$

(d) First, we need to determine the speed of the proton if the momentums are the same: $m_e v_e = m_p v_p$, so that

$$v_p = v_e\left(\frac{m_e}{m_p}\right) = (7.50\times10^6\ \text{m/s})\left(\frac{9.11\times10^{-31}\ \text{kg}}{1.67\times10^{-27}\ \text{kg}}\right) = \underline{4.09\times10^3\ \text{m/s}}$$

Then using $r = \frac{mv}{qB}$, we can determine the radius:

$$r = \frac{mv}{qB} = \frac{\left(1.67\times10^{-27}\ \text{kg}\right)\left(4.091\times10^3\ \text{m/s}\right)}{\left(1.60\times10^{-19}\ \text{C}\right)\left(1.00\times10^{-5}\ \text{T}\right)} = \underline{4.27\ \text{m}}$$

22.6 THE HALL EFFECT

25. *A nonmechanical water meter could utilize the Hall effect by applying a magnetic field across a metal pipe and measuring the Hall voltage produced. What is the average fluid velocity in a 3.00-cm-diameter pipe, if a 0.500-T field across it creates a 60.0-mV Hall voltage?*

Solution Using the equation $E = Blv$, we can determine the average velocity of the fluid. Note that the width is actually the diameter in this case:

$$v = \frac{E}{Bl} = \frac{60.0\times10^{-3}\ \text{V}}{(0.500\ \text{T})(0.0300\ \text{m})} = \underline{4.00\ \text{m/s}}$$

29. *Show that the Hall voltage across wires made of the same material, carrying identical currents, and subjected to the same magnetic field is inversely proportional to their diameters. (Hint: Consider how drift velocity depends on wire diameter.)*

Solution Using the equation $E = Blv$, where the width is twice the radius, $I = 2r$, and using the equation $I = nqAv_d$, we can get an expression for the drift velocity:

$$v_d = \frac{I}{nqA} = \frac{I}{nq\pi q^2}$$, so substituting into $E = Blv$, gives:

$$E = B \times 2r \times \frac{\mathrm{I}}{n\,q\,\pi\,q^2} = \frac{2IB}{nq\,\pi q} \propto \frac{1}{r} \propto \frac{1}{d}.$$

So, the Hall voltage is inversely proportional to the diameter of the wire.

22.7 MAGNETIC FORCE ON A CURRENT-CARRYING CONDUCTOR

36. *What force is exerted on the water in an MHD drive utilizing a 25.0-cm-diameter tube, if 100-A current is passed across the tube that is perpendicular to a 2.00-T magnetic field? (The relatively small size of this force indicates the need for very large currents and magnetic fields to make practical MHD drives.)*

Solution Using $F = IlB\sin\theta$, where l is the diameter of the tube, we can find the force on the water: $F = IlB\sin\theta = (100\,\mathrm{A})(0.250\,\mathrm{m})(2.00\,\mathrm{T})(1) = \underline{50.0\,\mathrm{N}}$

22.8 TORQUE ON A CURRENT LOOP: MOTORS AND METERS

42. *(a) What is the maximum torque on a 150-turn square loop of wire 18.0 cm on a side that carries a 50.0-A current in a 1.60-T field? (b) What is the torque when θ is $10.9°$?*

Solution (a) Using the equation $\tau_{\max} = NIAB\sin\phi$ we see that the maximum torque occurs when $\sin\phi = 1$, so the maximum torque is:

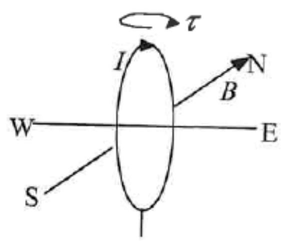
$$\tau_{\max} = NIAB\sin\phi = (150)(50.0\,\mathrm{A})(0.180\,\mathrm{m})^2(1.60\,\mathrm{T})(1) = \underline{389\,\mathrm{N\cdot m}}$$

(b) Now, use $\tau_{\max} = NIAB\sin\phi$, and set $\phi = 20.0°$, so that the torque is:

$$\tau = NIAB\sin\phi = (150)(50.0\,\mathrm{A})(0.180\,\mathrm{m})^2(1.60\,\mathrm{T})\sin 10.9° = \underline{73.5\,\mathrm{N\cdot m}}$$

48. *(a) A 200-turn circular loop of radius 50.0 cm is vertical, with its axis on an east-west line. A current of 100 A circulates clockwise in the loop when viewed from the east. The Earth's field here is due north, parallel to the ground, with a strength of* $3.00\times10^{-5}\ \text{T}$. *What are the direction and magnitude of the torque on the loop? (b) Does this device have any practical applications as a motcor?*

Solution



(a) The torque, τ, is clockwise as seen from directly above since the loop will rotate clockwise as seen from directly above. Using the equation $\tau_{\text{max}} = NIAB\sin\phi$, we find the maximum torque to be:

$$\tau = NIAB = (200)(100\ \text{A})\ \pi\ (0.500\ \text{m})^2\ (3.00\times10^{-5}\ \text{T}) = \underline{0.471\ \text{N}\cdot\text{m}}$$

(b) If the loop was connected to a wire, this is an example of a simple motor (see Figure 22.30). When current is passed through the loops, the magnetic field exerts a torque on the loops, which rotates a shaft. Electrical energy is converted to mechanical work in the process.

22.10 MAGNETIC FORCE BETWEEN TWO PARALLEL CONDUCTORS

50. *(a) The hot and neutral wires supplying DC power to a light-rail commuter train carry 800 A and are separated by 75.0 cm. What is the magnitude and direction of the force between 50.0 m of these wires? (b) Discuss the practical consequences of this force, if any.*

Solution

(a) Using the equation $\dfrac{F}{l} = \dfrac{\mu_0 I_1 I_2}{2\pi r}$, we can calculate the force on the wires:

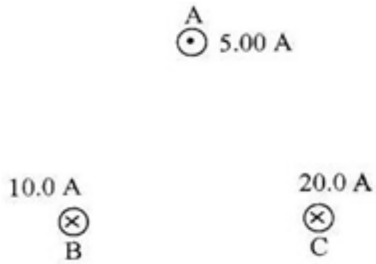
$$F = \frac{l\mu_0 I^2}{2\pi r} = \frac{(50.0)(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(800\ \text{A})^2}{2\pi(0.750\ \text{m})} = \underline{8.53\ \text{N}}$$

The force is <u>repulsive</u> because the currents are in opposite directions.

(b) This force is repulsive and therefore there is never a risk that the two wires will touch and short circuit.

56. *Find the direction and magnitude of the force that each wire experiences in Figure 22.58(a) by using vector addition.*

Solution



Opposites repel, likes attract, so we need to consider each wire's relationship with the other two wires. Let *f* denote force per unit length, then by

$$f = \frac{\mu_0 I_1 I_2}{2\pi r}$$

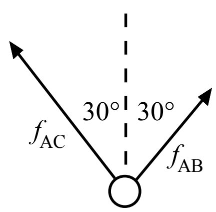
$$f_{AB} = \frac{(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A})(5.00\ \text{A})(10.0\ \text{A})}{2\pi(0.100\ \text{m})} = 1.00\times 10^{-4}\ \text{N/m}$$

$$f_{BC} = \frac{(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A})(10.0\ \text{A})(20.0\ \text{A})}{2\pi(0.100\ \text{m})} = 4.00\times 10^{-4}\ \text{N/m} = 4 f_{AB}$$

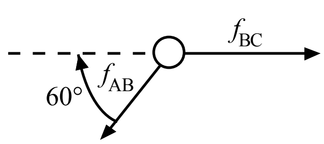
$$f_{AC} = \frac{(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A})(5.00\ \text{A})(20.0\ \text{A})}{2\pi(0.100\ \text{m})} = 2.00\times 10^{-4}\ \text{N/m} = 2 f_{AB}$$

Look at each wire separately:

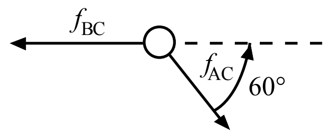
Wire A



Wire B

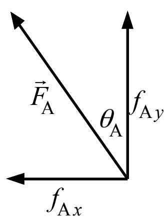


Wire C



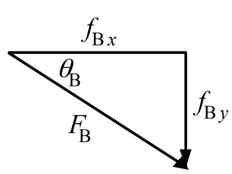
For wire A:

$$f_{Ax} = f_{AB}\sin 30° - f_{AC}\sin 30°$$
$$= \left(f_{AB} - 2f_{AB}\right)\sin 30° = -f_{AB}\left(\cos 30°\right) = -0.500\times 10^{-4}\ \text{N/m}$$
$$f_{Ay} = f_{AB}\cos 30° - f_{AC}\cos 30° = \left(f_{AB} - 2f_{AB}\right)\cos 30°$$
$$= -3f_{AB}\left(\cos 30°\right) = 2.60\times 10^{-4}\ \text{N/m}$$
$$F_A = \sqrt{f_{Ax}^2 + f_{Ay}^2} = \underline{2.65\times 10^{-4}\ \text{N/m}}$$
$$\theta_A = \tan^{-1}\left(\frac{|f_{Ax}|}{f_{Ay}}\right) = \underline{10.9°}$$



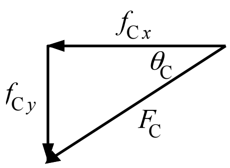
For Wire B:

$$f_{Bx} = f_{BC} - f_{AB}\cos 60°$$
$$= 4.00\times 10^{-4}\ \text{N/m} - \left(1.00\times 10^{-4}\ \text{N/m}\right)\cos 60° = 3.50\times 10^{-4}\ \text{N/m}$$
$$f_{By} = -f_{AB}\sin 60° = -\left(1.00\times 10^{-4}\ \text{N/m}\right)\sin 60° = -0.866\times 10^{-4}\ \text{N/m}$$
$$F_B = \sqrt{f_{Bx}^2 + f_{By}^2} = \underline{3.61\times 10^{-4}\ \text{N/m}}$$
$$\theta_B = \tan^{-1}\left(\frac{|f_{Bx}|}{f_{By}}\right) = \underline{13.9°}$$



For Wire C:

$$f_{Cx} = f_{AC}\cos 60° - f_{BC}$$
$$= \left(2.00\times10^{-4}\ \text{N/m}\right)\sin 60° - 4.00\times10^{-4}\ \text{N/m} = \underline{3.00\times10^{-4}\ \text{N/m}}$$
$$f_{Cy} = -f_{AC}\sin 60° - f_{BC}$$
$$= -\left(2.00\times10^{-4}\ \text{N/m}\right)\sin 60° - 4.00\times10^{-4}\ \text{N/m} = -1.73\times10^{-4}\ \text{N/m}$$
$$F_C = \sqrt{f_{Cx}^2 + f_{Cy}^2} = \underline{3.46\times10^{-4}\ \text{N/m}}$$
$$\theta_C = \tan^{-1}\left(\frac{f_{Cy}}{f_{Cx}}\right) = \underline{30.0°}$$



22.11 MORE APPLICATIONS OF MAGNETISM

77. ***Integrated Concepts*** *(a) Using the values given for an MHD drive in Exercise 22.36, and assuming the force is uniformly applied to the fluid, calculate the pressure created in* N/m^2*. (b) Is this a significant fraction of an atmosphere?*

Solution (a) Using $P = \dfrac{F}{A}$, we can calculate the pressure:

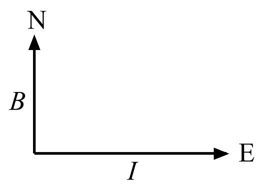
$$P = \frac{F}{A} = \frac{F}{\pi r^2} = \frac{50.0\ \text{N}}{\pi(0.125\ \text{m})^2} = \underline{1.02\times10^3\ \text{N/m}^2}$$

(b) No, this is not a significant fraction of an atmosphere.

$$\frac{P}{P_{\text{atm}}} = \frac{1.02\times10^3\ \text{N/m}^2}{1.013\times10^5\ \text{N/m}^2} = 1.01\%$$

83. ***Integrated Concepts*** *(a) What is the direction of the force on a wire carrying a current due east in a location where the Earth's field is due north? Both are parallel to the ground. (b) Calculate the force per meter if the wire carries 20.0 A and the field strength is* $3.00\times10^{-5}\ \text{T}$*. (c) What diameter copper wire would have its weight supported by this force? (d) Calculate the resistance per meter and the voltage per meter needed.*

Solution

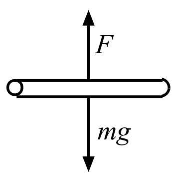


(a) Use the right hand rule-1. Put your right thumb to the east and your fingers to the north, then your palm points in the direction of the force, or up from the ground (out of the page).

(b) Using $F = IlB\sin\theta$, where $\theta = 90°$, so that $F = IlB\sin\theta$, or

$$\frac{F}{l} = IB\sin\theta = (20.0\ \text{A})(3.00\times10^{-5}\ \text{T})(1) = \underline{6.00\times10^{-4}\ \text{N/m}}$$

(c)



We want the force of the magnetic field to balance the weight force, so $F = mg$.

Now, to calculate the mass, recall $\rho = \dfrac{m}{V}$, where the volume is $\text{V} = \pi\,\text{r}^2 L$, so $\text{m} = \rho\text{V} = \rho\pi\,\text{r}^2 L$ and $F = \rho\pi\,\text{r}^2 Lg$, or

$$r = \sqrt{\frac{F/L}{\rho\pi g}} = \sqrt{\frac{6.00\times10^{-4}\ \text{N/m}}{(8.80\times10^3\ \text{kg/m}^3)(\pi)(9.80\ \text{m/s}^2)}} = 4.71\times10^{-5}\ \text{m}$$

$$\Rightarrow d = 2r = \underline{9.41\times10^{-5}\ \text{m}}$$

(d) From $R = \dfrac{\rho L}{A} = \dfrac{\rho L}{\pi\,\text{r}^2}$, where ρ is the resistivity:

$$\frac{R}{L} = \frac{\rho}{\pi\,\text{r}^2} = \frac{1.72\times10^{-8}\ \Omega.\text{m}}{\pi(4.71\times10^{-5}\ \text{m})^2} = \underline{2.47\ \Omega/\text{m}}.$$

Also, using the equation $I = \dfrac{V}{R}$, we find that

$$\frac{V}{L} = I\frac{R}{L} = (20.0\ \text{A})(2.47\ \Omega/\text{m}) = \underline{49.4\ \text{V/m}}$$

89. ***Unreasonable Results*** *A surveyor 100 m from a long straight 200-kV DC power line suspects that its magnetic field may equal that of the Earth and affect compass readings. (a) Calculate the current in the wire needed to create a* $5.00\times10^{-5}\ \text{T}$ *field at this distance. (b) What is unreasonable about this result? (c) Which assumption or premise is responsible?*

Solution (a) Using the equation $B=\dfrac{\mu_0 I}{2\pi\,\text{r}}$, we can calculate the current required to get the desired magnetic field strength:

$$I=\frac{(2\pi\,\text{r})B}{\mu_0}=\frac{2\pi(100\ \text{m})(5.00\times10^{-5}\ \text{T})}{4\pi\times10^{-7}\ \text{T.m/A}}=2.50\times10^{4}\ \text{A}=\underline{25.0\ \text{kA}}$$

(b) This current is unreasonably high. It implies a total power delivery in the line of $P=IV=(25.0\times10^{3}\ \text{A})(200\times10^{3}\ \text{V})=50.0\times10^{9}\ \text{W}=50.0\ \text{GW}$, which is much too high for standard transmission lines.

(c) 100 meters is a long distance to obtain the required field strength. Also coaxial cables are used for transmission lines so that there is virtually no field for DC power lines, because of cancellation from opposing currents. The surveyor's concerns are not a problem for his magnetic field measurements.

CHAPTER 23: ELECTROMAGNETIC INDUCTION, AC CIRCUITS, AND ELECTRICAL TECHNOLOGIES

23.1 INDUCED EMF AND MAGNETIC FLUX

1. *What is the value of the magnetic flux at coil 2 in Figure 23.56 due to coil 1?*

Solution Using the equation $\Phi = BA\cos\theta$, we can calculate the flux through coil 2, since the coils are perpendicular: $\Phi = BA\cos\theta = BA\cos 90° = \underline{0}$

23.2 FARADAY'S LAW OF INDUCTION: LENZ'S LAW

7. *Verify that the units of $\Delta\Phi / \Delta t$ are volts. That is, show that $1\ \text{T}\cdot\text{m}^2/\text{s} = 1\ \text{V}$.*

Solution The units of $\frac{\Delta\Phi}{\Delta t}$ will be:

$$\frac{[\Delta\Phi]}{[\Delta t]} = \frac{\text{T}\cdot\text{m}^2}{\text{s}} = (\text{N/A}\cdot\text{m})\left(\frac{\text{m}^2}{\text{s}}\right) = \frac{\text{N}\cdot\text{m}}{\text{A}\cdot\text{s}} = \frac{\text{N}\cdot\text{m}}{\text{C}} = \text{V so that } \underline{1\ \text{T}\cdot\text{m}^2/\text{s} = 1\ \text{V}}$$

14. *A lightning bolt produces a rapidly varying magnetic field. If the bolt strikes the earth vertically and acts like a current in a long straight wire, it will induce a voltage in a loop aligned like that in Figure 23.57(b). What voltage is induced in a 1.00 m diameter loop 50.0 m from a $2.00\times10^6\ \text{A}$ lightning strike, if the current falls to zero in $25.0\ \mu\text{s}$? (b) Discuss circumstances under which such a voltage would produce noticeable consequences.*

Solution (a) We know $E_0 = -\frac{N\Delta\Phi}{\Delta t}$, where the minus sign means that the emf creates the current and magnetic field that opposes the change in flux, and $\Phi = BA = (\pi r^2)B$.

Since the only thing that varies in the magnetic flux is the magnetic field, we can then say that $\Delta\Phi = \pi r^2 \Delta B$. Now, since $B = \dfrac{\mu_0 I}{2\pi d}$ the change in magnetic field occurs because of a change in the current, or $\Delta B = \dfrac{\mu_0 \Delta I}{2\pi d}$. Finally, substituting these into the equation $E_0 = -\dfrac{N\Delta\Phi}{\Delta t}$ gives:

$$E_0 = -\frac{N\pi r^2 \mu_0 \Delta I}{2\pi d \Delta t} = -\frac{\mu_0 N r^2 \Delta I}{2d\Delta t}$$

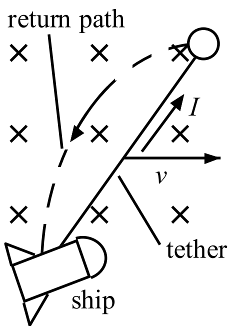
$$= -\frac{(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A})(1)(0.500\ \text{m})^2(-2.00\times 10^6\ \text{A})}{2(50.0\ \text{m})(2.50\times 10^{-5}\ \text{s})} = \underline{251\ \text{V}}$$

(b) An example is the alternator in your car. If you were driving during a lightning storm and this large bolt of lightning hit at 50.0 m away, it is possible to fry the alternator of your battery because of this large voltage surge. In addition, the hair at the back of your neck would stand on end because it becomes statically charged.

23.3 MOTIONAL EMF

16. *If a current flows in the Satellite Tether shown in Figure 23.12, use Faraday's law, Lenz's law, and RHR-1 to show that there is a magnetic force on the tether in the direction opposite to its velocity.*

Solution The flux through the loop (into the page) is increasing because the loop is getting larger and enclosing more magnetic field.



Thus, a magnetic field (out of the page) is induced to oppose the change in flux from the original field. Using RHR-2, point your fingers out of the page within the loop, then your thumb points in the counterclockwise direction around the loop, so the induced magnetic field is produced by the induction of a counterclockwise current in the circuit. Finally, using RHR-1, putting your right thumb in the direction of the current and your fingers into the page (in the direction of the magnetic field), your palm points to the left, so the magnetic force on the wire is to the left (in the direction opposite to its velocity).

23.4 EDDY CURRENTS AND MAGNETIC DAMPING

27. *A coil is moved through a magnetic field as shown in Figure 23.59. The field is uniform inside the rectangle and zero outside. What is the direction of the induced current and what is the direction of the magnetic force on the coil at each position shown?*

Solution (a) The magnetic field is zero and not changing, so there is no current and therefore no force on the coil.

(b) The magnetic field is increasing out of the page, so the induced magnetic field is into the page, created by an induced clockwise current. This current creates a force to the left.

(c) The magnetic field is not changing, so there is no current and therefore no force on the coil.

(d) The magnetic field is decreasing out of the page, so the induced magnetic field is out of the page, created by an induced counterclockwise current. This current creates a force to the right.

(e) The magnetic field is zero and not changing, so there is no current and therefore no force on the coil.

23.5 ELECTRIC GENERATORS

31. *What is the peak emf generated by a 0.250 m radius, 500-turn coil is rotated one-fourth of a revolution in 4.17 ms, originally having its plane perpendicular to a uniform magnetic field. (This is 60 rev/s.)*

Solution Using the information given in Exercise 23.12:

$$\Delta\theta = \frac{1}{4}\,\text{rev} = \frac{1}{4}(2\pi\ \text{rad})\,\text{and}\ \Delta t = 4.17\times10^{-3}\ \text{s},$$

$$N = 500;\ A = \pi r^2 = \pi(0.250\ \text{m})^2;\ \text{and}\ B = 0.425\ \text{T},$$

we get: $\omega = \dfrac{\Delta\theta}{\Delta t} = \dfrac{(1/4)(2\pi)\,\text{rad}}{4.17\times10^{-3}\ \text{s}} = 376.7\ \text{rad/s}$. Therefore,

$$E_0 = NAB\omega = (500)(\pi)(0.250\ \text{m})^2(0.425\ \text{T})(376.7\ \text{rad/s}) = 1.57\times10^4\ \text{V} = \underline{15.7\ \text{kV}}$$

23.6 BACK EMF

39. *Suppose a motor connected to a 120 V source draws 10.0 A when it first starts. (a) What is its resistance? (b) What current does it draw at its normal operating speed when it develops a 100 V back emf?*

Solution (a) Using the equation $I = \dfrac{V}{R}$, we can determine given the voltage and the current:

$$R = \frac{V}{I} = \frac{120\ \text{V}}{20.0\ \text{A}} = \underline{6.00\ \Omega}$$

(b) Again, using $I = \dfrac{V}{R}$, we can now determine the current given that the net voltage is the difference between the source voltage and the back emf:

$$I = \frac{V}{R} = \frac{120\ \text{V} - 100\ \text{V}}{6.00\ \Omega} = \underline{3.33\ \text{A}}$$

43. *The motor in a toy car is powered by four batteries in series, which produce a total emf of 6.00 V. The motor draws 3.00 A and develops a 4.50 V back emf at normal speed. Each battery has a* $0.100\,\Omega$ *internal resistance. What is the resistance of the motor?*

Solution Since the resistors are in series, we know the total internal resistance of the batteries is $R = 4(0.100\ \Omega)$. Therefore,

$$I = \frac{E-V}{R+R'},\ \text{so that}\ R + R' = \frac{E-V}{I} \Rightarrow R' = \frac{6.00\ \text{V} - 4.50\ \text{V}}{3.00\ \text{A}} - 4(0.100\ \Omega) = \underline{0.100\ \ \Omega}$$

23.7 TRANSFORMERS

46. *A cassette recorder uses a plug-in transformer to convert 120 V to 12.0 V, with a maximum current output of 200 mA. (a) What is the current input? (b) What is the power input? (c) Is this amount of power reasonable for a small appliance?*

Solution (a) Using the equations $\frac{V_s}{V_p} = \frac{N_s}{N_p}$ and $\frac{V_s}{V_p} = \frac{I_p}{I_s}$, we can determine the primary current: $\frac{I_p}{I_s} = \frac{N_p}{N_s} = \frac{V_p}{V_s}$, so that

$$I_p = I_s\left(\frac{V_p}{V_s}\right) = (0.200\ \text{A})\left(\frac{12.0\ \text{V}}{120\ \text{V}}\right) = 2.00\times10^{-2}\ \text{A} = \underline{20.0\ \text{mA}}$$

(b) $P_{in} = I_p V_p = (0.200\ \text{A})120\ \text{V} = \underline{2.40\ \text{W}}$

(c) Yes, this amount of power is quite reasonable for a small appliance.

23.9 INDUCTANCE

55. *Two coils are placed close together in a physics lab to demonstrate Faraday's law of induction. A current of 5.00 A in one is switched off in 1.00 ms, inducing a 9.00 V emf in the other. What is their mutual inductance?*

Solution Using the equation $E_2 = -M\frac{\Delta I_1}{\Delta t}$, where the minus sign is an expression of Lenz's law, we can calculate the mutual inductance between the two coils:

$$M = E_2\frac{\Delta t}{\Delta I_1} = (9.00\ \text{V})\frac{(1.00\times10^{-3}\ \text{s})}{5.00\ \text{A}} = \underline{1.80\ \text{mH}}$$

61. *A large research solenoid has a self-inductance of 25.0 H. (a) What induced emf opposes shutting it off when 100 A of current through it is switched off in 80.0 ms? (b) How much energy is stored in the inductor at full current? (c) At what rate in watts must energy be dissipated to switch the current off in 80.0 ms? (d) In view of the answer to the last part, is it surprising that shutting it down this quickly is difficult?*

Solution (a) Using the equation $E = L\frac{\Delta I}{\Delta t}$, we have

$$E = L\frac{\Delta I}{\Delta t} = (25.0\ \text{H})\frac{(100\ \text{A})}{(8.00\times10^{-2}\ \text{s})} = 3.125\times10^{-4}\ \text{V} = \underline{31.3\ \text{kV}}$$

(b) Using $E_{\text{ind}} = \frac{1}{2}LI^2 = \left(\frac{1}{2}\right)(25.0\ \text{H})(100\ \text{A})^2 = \underline{1.25\times10^5\ \text{J}}$

(c) Using the equation $P = \frac{\Delta E}{\Delta t}$, we have

$$P = \frac{\Delta E}{\Delta t} = \frac{1.25\times10^5\ \text{J}}{8.00\times10^{-2}\ \text{s}} = 1.563\times10^6\ \text{W} = \underline{1.56\ \text{MW}}$$

(d) <u>No</u>, it is not surprising since this power is very high.

68. ***Unreasonable Results*** *A 25.0 H inductor has 100 A of current turned off in 1.00 ms. (a) What voltage is induced to oppose this? (b) What is unreasonable about this result? (c) Which assumption or premise is responsible?*

Solution (a) $|E| = L\frac{\Delta I}{\Delta t} = (25.0\ \text{H})\frac{(100\ \text{A})}{1.00\times10^{-3}\ \text{s}} = \underline{2.50\times10^6\ \text{V}}$

(b) The voltage is so extremely high that arcing would occur and the current would not be reduced so rapidly.

(c) It is not reasonable to shut off such a large current in such a large inductor in such an extremely short time.

23.10 RL CIRCUITS

69. *If you want a characteristic RL time constant of 1.00 s, and you have a $500\,\Omega$ resistor, what value of self-inductance is needed?*

Solution Using the equation $\tau = \frac{L}{R}$, we know $L = \tau R = (1.00\ \text{s})(500\ \Omega) = \underline{500\ \text{H}}$

75. *What percentage of the final current I_0 flows through an inductor L in series with a resistor R, three time constants after the circuit is completed?*

Solution We use the equation $I = I_0\left(1-e^{-t/\tau}\right)$, because the problem says, "after the circuit is completed." Thus, the final current is given by: $I = I_0\left(1-e^{-t/\tau}\right)$, where $t = 3\tau$ so that:

$$\frac{I}{I_0} = \left(1-e^{-t/\tau}\right) = 1-e^{-3} = 0.9502$$

The current is <u>95.0%</u> of the final current after 3 time constants.

23.11 REACTANCE, INDUCTIVE AND CAPACITIVE

81. *What capacitance should be used to produce a* $2.00\ \text{M}\Omega$ *reactance at 60.0 Hz?*

Solution Using the equation $X_C = \dfrac{1}{2\pi fC}$, we can determine the necessary capacitance:

$$C = \frac{1}{2\pi f X_C} = \frac{1}{2\pi\,(60.0\ \text{Hz})(2.00\times10^6\ \Omega)} = 1.326\times10^{-9}\ \text{F} = \underline{1.33\ \text{nF}}$$

87. *(a) An inductor designed to filter high-frequency noise from power supplied to a personal computer is placed in series with the computer. What minimum inductance should it have to produce a* $2.00\ \text{k}\Omega$ *reactance for 15.0 kHz noise? (b) What is its reactance at 60.0 Hz?*

Solution (a) Using the equation $X_L = 2\pi fL$,

$$X_L = 2\pi fL,\ \text{or}\ L = \frac{X_L}{2\pi f} = \frac{(2.00\times10^3\ \Omega)}{2\pi\,(1.50\times10^4\ \text{Hz})} = 2.122\times10^{-2}\ \text{H} = \underline{21.2\ \text{mH}}$$

(b) Again using $X_L = 2\pi fL$,

$$X_L = 2\pi fL = 2\pi(60.0\ \text{Hz})(2.122\times10^{-2}\ \text{H}) = \underline{8.00\ \ \Omega}$$

23.12 RLC SERIES AC CIRCUITS

95. What is the resonant frequency of a 0.500 mH inductor connected to a $40.0\ \mu\text{F}$ capacitor?

Solution Using the equation $f_0 = \dfrac{1}{2\pi\sqrt{LC}}$, we can determine the resonant frequency for the circuit: $f_0 = \dfrac{1}{2\pi\sqrt{LC}} = \dfrac{1}{2\pi\sqrt{(5.00\times10^{-4}\text{ H})(4.00\times10^{-5}\text{ F})}} = 1.125\times10^{3}\text{ Hz} = \underline{1.13\text{ kHz}}$

101. *An RLC series circuit has a* $2.50\ \Omega$ *resistor, a* $100\ \mu\text{H}$ *inductor, and an* $80.0\ \mu\text{F}$ *capacitor. (a) Find the circuit's impedance at 120 Hz. (b) Find the circuit's impedance at 5.00 kHz. (c) If the voltage source has* $V_{\text{rms}} = 5.60\text{ V}$*, what is* I_{rms} *at each frequency? (d) What is the resonant frequency of the circuit? (e) What is* I_{rms} *at resonance?*

Solution (a) The equation $X_L = 2\pi fL$ gives the inductive reactance:

$$X_L = 2\pi fL = 2\pi(120\text{ Hz})(1.00\times10^{-4}\text{ H}) = 7.540\times10^{-2}\ \Omega$$

The equation $X_C = \dfrac{1}{2\pi fC}$ gives the capacitive reactance:

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi(120\text{ Hz})(8.00\times10^{-5}\text{ F})} = 16.58\ \Omega$$

Finally, the equation $Z = \sqrt{R^2 + (X_L - X_C)^2}$ gives the impedance of the RLC circuit:

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(2.50\ \Omega)^2 + (7.54\times10^{-2}\ \Omega - 16.58\ \Omega)^2} = \underline{16.7\ \Omega}$$

(b) Again, $X_L = 2\pi fL$ gives the inductive reactance:

$$X_L = 2\pi(5.00\times10^{3}\text{ Hz})(1.00\times10^{-4}\text{ H}) = 3.142\ \Omega$$

$X_C = \dfrac{1}{2\pi fC}$ gives the capacitive reactance:

$$X_C = \frac{1}{2\pi(5.00\times10^{3}\text{ Hz})(8.00\times10^{-4}\text{ F})} = 3.979\times10^{-1}\ \Omega$$

And $Z = \sqrt{R^2 + (X_L - X_C)^2}$ gives the impedance:

$$Z = \sqrt{(2.50\ \Omega)^2 + (3.142\ \Omega - 3.979\times10^{-1}\ \Omega)^2} = \underline{3.71\ \Omega}$$

(c) The rms current is found using the equation $I_{\text{rms}} = \frac{V_{\text{rms}}}{R}$. For $f = 120\ \text{Hz}$,

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{5.60\ \text{V}}{16.69\ \Omega} = \underline{0.336\ \text{A}}$$

and for $f = 5.00\ \text{kHz}$, $I_{\text{rms}} = \frac{5.60\ \text{V}}{3.712\ \Omega} = \underline{1.51\ \text{A}}$

(d) The resonant frequency is found using the equation $f_0 = \frac{1}{2\pi\sqrt{LC}}$:

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(1.00\times10^{-7}\ \text{H})(8.00\times10^{-5}\ \text{F})}} = 5.63\times10^{4}\ \text{Hz} = \underline{56.3\ \text{kHz}}$$

(e) At resonance, $X_L = X_R$, so that $Z = R$ and $I_{\text{rms}} = \frac{V_{\text{rms}}}{R}$ reduces to:

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{R} = \frac{5.60\ \text{V}}{2.50\ \Omega} = \underline{2.24\ \text{A}}$$

CHAPTER 24: ELECTROMAGNETIC WAVES

24.1 MAXWELL'S EQUATIONS: ELECTROMAGNETIC WAVES PREDICTED AND OBSERVED

1. *Verify that the correct value for the speed of light c is obtained when numerical values for the permeability and permittivity of free space (μ_0 and ε_0) are entered into the equation $c=\dfrac{1}{\sqrt{\mu_0\varepsilon_0}}$.*

Solution We know that $\mu_0 = 4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A}$, and from Section 19.1, we know that $\varepsilon_0 = 8.85\times 10^{-12}\ \text{F/m}$, so that the equation becomes:

$$c=\frac{1}{\sqrt{\left(4\pi\times 10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(8.8542\times 10^{-12}\ \text{F/m}\right)}}=2.998\times 10^{8}\ \text{m/s}=\underline{3.00\times 10^{8}\ \text{m/s}}$$

The units work as follows:

$$[c]=\frac{1}{\sqrt{\text{T}\cdot\text{F/A}}}=\sqrt{\frac{\text{A}}{\text{T}\cdot\text{F}}}=\sqrt{\frac{\text{C/s}}{(\text{N}\cdot\text{s/C}\cdot\text{m})\times(\text{C}^2/\text{J})}}=\sqrt{\frac{\text{J}\cdot\text{m}}{\text{N}\cdot\text{s}^2}}=\sqrt{\frac{(\text{N}\cdot\text{m})\text{m}}{\text{N}\cdot\text{s}^2}}=\sqrt{\frac{\text{m}^2}{\text{s}^2}}=\text{m/s}$$

24.3 THE ELECTROMAGNETIC SPECTRUM

8. *A radio station utilizes frequencies between commercial AM and FM. What is the frequency of a 11.12-m-wavelength channel?*

Solution Using the equation $c=f\lambda$, we can solve for the frequency since we know the speed of light and are given the wavelength;

$$f=\frac{c}{\lambda}=\frac{2.998\times 10^{8}\ \text{m/s}}{11.2\ \text{m}}=2.696\times 10^{7}\ \text{s}^{-1}=\underline{26.96\ \text{MHz}}$$

17. *If the Sun suddenly turned off, we would not know it until its light stopped coming. How long would that be, given that the Sun is $1.50\times 10^{11}\ \text{m}$ away?*

Solution We know that $v=\frac{d}{t}$, and since we know the speed of light and the distance from the sun to the earth, we can calculate the time: $t=\frac{d}{c}=\frac{1.50\times10^{11}\text{ m}}{3.00\times10^{8}\text{ m/s}}=\underline{500\text{ s}}$

23. *(a) What is the frequency of the 193-nm ultraviolet radiation used in laser eye surgery? (b) Assuming the accuracy with which this EM radiation can ablate the cornea is directly proportional to wavelength, how much more accurate can this UV be than the shortest visible wavelength of light?*

Solution (a) Using the equation $c = f\lambda$ we can calculate the frequency given the speed of light and the wavelength of the radiation:

$$f=\frac{c}{\lambda}=\frac{3.00\times10^{8}\text{ m/s}}{1.93\times10^{-7}\text{ m}}=1.55\times10^{15}\text{ s}^{-1}=\underline{1.55\times10^{15}\text{ Hz}}$$

(b) The shortest wavelength of visible light is 380 nm, so that: $\frac{\lambda_{\text{visible}}}{\lambda_{\text{UV}}}=\frac{380\text{ nm}}{193\text{ nm}}=1.97.$

In other words, the UV radiation is 97% more accurate than the shortest wavelength of visible light, or almost twice as accurate.

24.4 ENERGY IN ELECTROMAGNETIC WAVES

31. *Find the intensity of an electromagnetic wave having a peak magnetic field strength of* $4.00\times10^{-9}\text{ T}$.

Solution Using the equation $I_{\text{ave}}=\frac{cB_0^{\,2}}{2\mu_0}$ we see that:

$$I_{\text{ave}}=\frac{cB_0^{\,2}}{2\mu_0}=\frac{(3.00\times10^{8}\text{ m/s})(4.00\times10^{-9}\text{ T})^2}{2(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})}=\underline{1.91\times10^{-3}\text{ W/m}^2}$$

The units work as follows:

$$[I]=\frac{(\text{m/s})\text{T}^2}{\text{T}\cdot\text{m/A}}=\frac{\text{T}\cdot\text{A}}{\text{s}}=\frac{(\text{N/A}\cdot\text{m})(\text{A})}{\text{s}}=\frac{\text{N}}{\text{s.m}}=\frac{\text{J/m}}{\text{s}\cdot\text{m}}=\frac{\text{W}}{\text{m}^2}$$

36. *Lasers can be constructed that produce an extremely high intensity electromagnetic wave for a brief time—called pulsed lasers. They are used to ignite nuclear fusion, for example. Such a laser may produce an electromagnetic wave with a maximum electric field strength of* $1.00\times10^{11}\ \text{V/m}$ *for a time of 1.00 ns. (a) What is the maximum magnetic field strength in the wave? (b) What is the intensity of the beam? (c) What energy does it deliver on a* 1.00-mm^2 *area?*

Solution (a) Using the equation $\frac{E}{B}=c$, we can determine the maximum magnetic field strength given the maximum electric field strength:

$B_0=\frac{E_0}{c}=\frac{1.00\times10^{11}\ \text{N/C}}{3.00\times10^{8}\ \text{m/s}}=\underline{333\ \text{T}}$, recalling that 1 V/m = 1 N/C.

(b) Using the equation $I_{\text{ave}}=\frac{c\varepsilon_0E_0^2}{2}$, we can calculate the intensity without using the result from part (a):

$$I=\frac{c\varepsilon_0E_0^2}{2}$$

$$=\frac{(3.00\times10^{8}\ \text{m/s})(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m})(1.00\times10^{11}\ \text{N/C})^2}{2}=\underline{1.33\times10^{19}\ \text{W/m}^2}$$

(c) We can get an expression for the power in terms of the intensity: $P=IA$, and from the equation $E=Pt$, we can express the energy in terms of the power provided. Since we are told the time of the laser pulse, we can calculate the energy delivered to a $1.00\ \text{mm}^2$ area per pulse:

$$E=P\Delta t=IA\Delta t$$

$$=(1.328\times10^{19}\ \text{W/m}^2)(1.00\ \text{mm}^2)\left(\frac{1\ \text{m}}{1000\ \text{mm}}\right)^2(1.00\times10^{-9}\ \text{s})$$

$$=1.33\times10^{4}\ \text{J}=\underline{13.3\ \text{kJ}}$$

40. ***Integrated Concepts*** *What capacitance is needed in series with an* $800\text{-}\mu\text{H}$ *inductor to form a circuit that radiates a wavelength of 196 m?*

Solution Using the equation $f_0=\frac{1}{2\pi\sqrt{LC}}$, we can find the capacitance in terms of the resonant frequency: $C=\frac{1}{4\pi^2 L f_0^2}$. Substituting for the frequency, using the equation

$c=f\lambda$ gives: $C=\frac{\lambda^2}{4\pi^2 Lc^2}\frac{(196\ \text{m})^2}{4\pi^2\left(8.00\times10^{-4}\ \text{H}\right)\left(3.00\times10^8\ \text{m/s}\right)}=1.35\times10^{-11}\ \text{F}=\underline{13.5\ \text{pF}}$

The units work as follows: $[C]=\frac{\text{m}^2}{\text{H}(\text{m/s})^2}=\frac{\text{s}^2}{\text{H}}=\frac{\text{s}^2}{\Omega\cdot\text{s}}=\frac{\text{s}}{\Omega}=\frac{\text{s}}{\text{V/A}}=\frac{\text{A}\cdot\text{s}}{\text{V}}=\frac{\text{C}}{\text{V}}=\text{F}$

44. ***Integrated Concepts*** *Electromagnetic radiation from a 5.00-mW laser is concentrated on a* 1.00-mm^2 *area. (a) What is the intensity in* W/m^2*? (b) Suppose a 2.00-nC static charge is in the beam. What is the maximum electric force it experiences? (c) If the static charge moves at 400 m/s, what maximum magnetic force can it feel?*

Solution

(a) From the equation $I=\frac{P}{A}$, we know: $I=\frac{P}{A}=\frac{5.00\times10^{-3}\ \text{W}}{1.00\times10^{-6}\ \text{m}^2}=\underline{5.00\times10^3\ \text{W/m}^2}$

(b) Using the equation $I_{\text{ave}}=\frac{c\varepsilon_0 E_0^{\ 2}}{2}$, we can solve for the maximum electric field:

$$E_0=\sqrt{\frac{2I}{c\varepsilon_0}}=\sqrt{\frac{2(5.00\times10^3\ \text{W/m}^2)}{(3.00\times10^8\ \text{m/s})(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)}}=1.94\times10^3\ \text{N/C}.$$

So, using the equation $E=\frac{F}{q}$ we can calculate the force on a 2.00 nC charges:

$$F=qE_0=\left(2.00\times10^{-9}\ \text{C}\right)\left(1.94\times10^3\ \text{N/C}\right)=\underline{3.88\times10^{-6}\ \text{N}}$$

(c) Using the equations $F=qvB\sin\theta$ and $\frac{E}{B}=c$, we can write the maximum magnetic force in terms of the electric field, since the electric and magnetic fields are related for electromagnetic radiation:

$$F_{B,\max}=qvB_0=\frac{qvE_0}{c}=\frac{\left(2.00\times10^{-9}\ \text{C}\right)\left(400\ \text{m/s}\right)\left(1.94\times10^3\ \text{N/C}\right)}{3.00\times10^8\ \text{m/s}}=\underline{5.18\times10^{-12}\ \text{N}}$$

So the electric force is approximately 6 orders of magnitude stronger than the magnetic force.

50. ***Unreasonable Results*** *An LC circuit containing a 1.00-pF capacitor oscillates at such a frequency that it radiates at a 300-nm wavelength. (a) What is the inductance of the circuit? (b) What is unreasonable about this result? (c) Which assumptions are unreasonable or inconsistent?*

Solution (a) Using the equations $f_0 = \dfrac{1}{2\pi\sqrt{LC}}$ and $c = f\lambda$, we can solve for the inductance:

$f = \dfrac{c}{\lambda} = \dfrac{1}{2\pi\sqrt{LC}}$, so that

$$L = \frac{\lambda^2}{4\pi^2 Cc^2} = \frac{\left(3.00\times10^{-7}\ \text{m}\right)^2}{4\pi^2\left(1.00\times10^{-12}\ \text{F}\right)\left(3.00\times10^{8}\ \text{m/s}\right)^2} = \underline{2.53\times10^{-20}\ \text{H}}$$

(b) This inductance is unreasonably small.

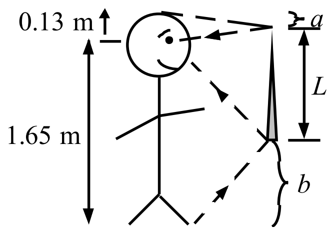
(c) The wavelength is too small.

CHAPTER 25: GEOMETRIC OPTICS

25.1 THE RAY ASPECT OF LIGHT

1. *Suppose a man stands in front of a mirror as shown in Figure 25.50. His eyes are 1.65 m above the floor, and the top of his head is 0.13 m higher. Find the height above the floor of the top and bottom of the smallest mirror in which he can see both the top of his head and his feet. How is this distance related to the man's height?*

Solution



From ray-tracing and the law of reflection, we know that the angle of incidence is equal to the angle of reflection, so the top of the mirror must extend to at least halfway between his eyes and the top of his head. The bottom must go down to halfway between his eyes and the floor. This result is independent of how far he stands from the wall. Therefore, $a=\dfrac{0.13\text{ m}}{2}=0.065\text{ m}$, $b=\dfrac{1.65\text{ m}}{2}=0.825\text{ m}$ and

$$L=1.65\text{ m}+0.13\text{ m}-a-b=1.78\text{ m}-\frac{0.13\text{ m}}{2}-\frac{1.65\text{ m}}{2}=0.89\text{ m}$$

The bottom is $b=\underline{0.825\text{ m}}$ from the floor and the top is $b+L=0.825\text{ m}+0.89\text{ m}=\underline{1.715\text{ m}}$ from the floor.

25.3 THE LAW OF REFRACTION

7. *Calculate the index of refraction for a medium in which the speed of light is* $2.012\times10^{8}\text{ m/s}$*, and identify the most likely substance based on Table 25.1.*

Solution

Use the equation $n=\dfrac{c}{v}=\dfrac{2.997\times10^{8}\text{ m/s}}{2.012\times10^{8}\text{ m/s}}=\underline{1.490}$. From Table 25.1, the substance is polystyrene.

13. *Suppose you have an unknown clear substance immersed in water, and you wish to identify it by finding its index of refraction. You arrange to have a beam of light enter it at an angle of* $45.0°$*, and you observe the angle of refraction to be* $40.3°$*. What is the index of refraction of the substance and its likely identity?*

Solution Using the equation $n_1 \sin\theta_1 = n_2 \sin\theta$, we can solve for the unknown index of refraction: $n_2 = n_1 = \frac{\sin\theta_1}{\sin\theta_2} = \frac{(1.33)(\sin 45.0°)}{\sin 40.3°} = \underline{1.46}$

From Table 25.1, the most likely solid substance is fused quartz.

25.4 TOTAL INTERNAL REFLECTION

22. *An optical fiber uses flint glass clad with crown glass. What is the critical angle?*

Solution Using the equation $\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$ and the indices of refraction from Table 25.1 gives a critical angle of $\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right) = \sin^{-1}\left(\frac{1.52}{1.66}\right) = \underline{66.3°}$

25.5 DISPERSION: THE RAINBOW AND PRISMS

33. *A ray of 610 nm light goes from air into fused quartz at an incident angle of* $55.0°$*. At what incident angle must 470 nm light enter flint glass to have the same angle of refraction?*

Solution Using Snell's law, we have $n_1 \sin\theta_1 = n_2 \sin\theta_2$ and $n'_1 \sin\theta'_1 = n'_2 \sin\theta'_2$. We can set θ_2 equal to θ'_2, because the angles of refraction are equal. Combining the equations gives $\frac{n_1 \sin\theta_1}{n_2} = \frac{n'_1 \sin\theta'_1}{n'_2}$. We know that $n_1 = n'_1 = 1.00$ because the light is entering from air. From Table 25.2, we find the 610 nm light in fused quartz has $n_2 = 1.456$ and the 470 nm light in flint glass has $n'_2 = 1.684$. We can solve for the incident angle θ_1:

$$\theta'_1 = \sin^{-1}\left(\frac{n_1 n'_2}{n_2 n'_1}\sin\theta_1\right) = \sin^{-1}\left[\frac{(1)(1.684)}{(1.456)(1)}\sin 55.0°\right] = \underline{71.3°}$$

25.6 IMAGE FORMATION BY LENSES

39. *You note that your prescription for new eyeglasses is –4.50 D. What will their focal length be?*

Solution Using the equation $P=\frac{1}{f}$, we can solve for the focal length of your eyeglasses, recalling that $1\,\text{D}=1/\text{m}$:

$$f=\frac{1}{P}=\frac{1}{-4.50\ \text{D}}=-0.222\ \text{m}=\underline{-22.2\ \text{cm}}.$$

43. *How far from a piece of paper must you hold your father's 2.25 D reading glasses to try to burn a hole in the paper with sunlight?*

Solution Using the equation $P=\frac{1}{f}$, we can solve for the focal length for your father's reading glasses: $f=\frac{1}{P}=\frac{1}{2.25\ \text{D}}=0.444\ \text{m}=\underline{44.4\ \text{cm}}$. In order to burn a hole in the paper, you want to have the glasses exactly one focal length from the paper, so the glasses should be 44.4 cm from the paper.

49. *In Example 25.7, the magnification of a book held 7.50 cm from a 10.0 cm focal length lens was found to be 3.00. (a) Find the magnification for the book when it is held 8.50 cm from the magnifier. (b) Do the same for when it is held 9.50 cm from the magnifier. (c) Comment on the trend in m as the object distance increases as in these two calculations.*

Solution (a) Using the equation $\frac{1}{d_\text{o}}+\frac{1}{d_\text{i}}=\frac{1}{f}$, we can first determine the image distance:

$$d_\text{i}=\left(\frac{1}{f}-\frac{1}{d_\text{o}}\right)^{-1}=\left(\frac{1}{10.0\ \text{cm}}-\frac{1}{8.50\ \text{cm}}\right)^{-1}=-56.67\text{cm}.$$ Then we can determine the magnification using the equation $m=-\frac{d_\text{i}}{d_\text{o}}=\frac{56.67\ \text{cm}}{8.50\ \text{cm}}=\underline{6.67}$.

(b) Using this equation again gives: $d_\text{i}=\left(\frac{1}{10.0\ \text{cm}}-\frac{1}{9.50\ \text{cm}}\right)^{-1}=-190\ \text{cm}$

And a magnification of $m = -\frac{d_\text{i}}{d_\text{o}} = \frac{190\text{ cm}}{9.5\text{ cm}} = \underline{+20.0}$

(c) The magnification, m, increases rapidly as you increase the object distance toward the focal length.

25.7 IMAGE FORMATION BY MIRRORS

57. *What is the focal length of a makeup mirror that produces a magnification of 1.50 when a person's face is 12.0 cm away? Explicitly show how you follow the steps in the Problem-Solving Strategy for Mirrors.*

Solution *Step 1*: Image formation by a mirror is involved.

Step 2: Draw the problem set up when possible.

Step 3: Use the thin lens equations to solve this problem.

Step 4: Find f.

Step 5: Given: $m = 1.50, d_\text{o} = 0.120\text{ m}$.

Step 6: No ray tracing is needed.

Step 7: Using the equation $m = -\frac{d_\text{i}}{d_\text{o}}$, we know that

$d_\text{i} = -md_\text{o} = -(1.50)(0.120\text{ m}) = -0.180\text{ m}$. Then, using the equation $\frac{1}{d_\text{o}} + \frac{1}{d_i} = \frac{1}{f}$, we can determine the focal length: $f = \left(\frac{1}{d_\text{i}} + \frac{1}{d_\text{o}}\right)^{-1} = \left(\frac{1}{-0.180\text{ m}} + \frac{1}{0.120\text{ m}}\right)^{-1} = \underline{0.360\text{ m}}$

Step 8: Since the focal length is greater than the object distance, we are dealing with case 2. For case 2, we should have a virtual image, a negative image distance and a positive (greater than one) magnification. Our answer is consistent with these expected properties, so it is reasonable.

CHAPTER 26: VISION AND OPTICAL INSTRUMENTS

26.1 PHYSICS OF THE EYE

2. *Calculate the power of the eye when viewing an object 3.00 m away.*

Solution Using the lens-to-retina distance of 2.00 cm and the equation $P=\frac{1}{d_o}+\frac{1}{d_i}$ we can determine the power at an object distance of 3.00 m:

$$P=\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{3.00\text{ m}}+\frac{1}{0.0200\text{ m}}=\underline{+50.3\text{ D}}$$

26.2 VISION CORRECTION

10. *What was the previous far point of a patient who had laser vision correction that reduced the power of her eye by 7.00 D, producing normal distant vision for her?*

Solution Since normal distant vision has a power of 50.0 D (Example 26.2) and the laser vision correction reduced the power of her eye by 7.00 D, she originally had a power of 57.0 D. We can determine her original far point using

$$P=\frac{1}{d_o}+\frac{1}{d_i}\Rightarrow d_o=\left(P-\frac{1}{d_i}\right)^{-1}=\left(57.0\text{ D}-\frac{1}{0.0200\text{ m}}\right)^{-1}=\underline{0.143\text{ m}}$$

Originally, without corrective lenses, she could only see images 14.3 cm (or closer) to her eye.

14. *A young woman with normal distant vision has a 10.0% ability to accommodate (that is, increase) the power of her eyes. What is the closest object she can see clearly?*

Solution From Example 26.2, the normal power for distant vision is 50.0 D. For this woman, since she has a 10.0% ability to accommodate, her maximum power is $P_{\text{max}} = (1.10)(50.0\ \text{D}) = 55.0\ \text{D}$. Thus using the equation $P = \frac{1}{d_o} + \frac{1}{d_i}$, we can determine the nearest object she can see clearly since we know the image distance must be the lens-to-retina distance of 2.00cm:

$$P = \frac{1}{d_o} + \frac{1}{d_i} \Rightarrow d_o = \left(P - \frac{1}{d_i}\right)^{-1} = \left(55.0\ \text{D} - \frac{1}{0.0200\ \text{m}}\right)^{-1} = 0.200\ \text{m} = \underline{20.0\ \text{cm}}$$

26.5 TELESCOPES

37. *A $7.5\times$ binocular produces an angular magnification of -7.50, acting like a telescope. (Mirrors are used to make the image upright.) If the binoculars have objective lenses with a 75.0 cm focal length, what is the focal length of the eyepiece lenses?*

Solution Using the equation $M = -\frac{f_o}{f_e}$, we can determine the focal length of the eyepiece since we know the magnification and the focal length of the objective:

$$f_e = -\frac{f_o}{M} = -\frac{75.0\ \text{cm}}{-7.50} = \underline{+10.0\ \text{cm}}$$

26.6 ABERRATIONS

39. ***Integrated Concepts*** *(a) During laser vision correction, a brief burst of 193 nm ultraviolet light is projected onto the cornea of the patient. It makes a spot 1.00 mm in diameter and deposits 0.500 mJ of energy. Calculate the depth of the layer ablated, assuming the corneal tissue has the same properties as water and is initially at* $34.0°\text{C}$. *The tissue's temperature is increased to* $100°\text{C}$ *and evaporated without further temperature increase. (b) Does your answer imply that the shape of the cornea can be finely controlled?*

Solution (a) We can get an expression for the heat transfer in terms of the mass of tissue ablated: $Q = mc\Delta T + mL_v = m(c\Delta T + L_v)$, where the heat capacity is given in Table 14.1, $c = 4186\ \text{J/kg}\cdot°\text{C}$, and the latent heat of vaporization is given in Table 14.2, $L_v = 2256\times 10^3\ \text{J/kg}$. Solving for the mass gives:

$$m = \frac{Q}{c\Delta T + L_\text{v}}$$

$$= \frac{(5.00\times10^{-4}\ \text{J})}{(4186\ \text{J/kg}\cdot{}^\circ\text{C})(100^\circ\ \text{C} - 34.0^\circ\ \text{C}) + 2.256\times10^{6}\ \text{J/kg}} = 1.975\times10^{-10}\ \text{kg}$$

Now, since the corneal tissue has the same properties as water, its density is $1000\ \text{kg/m}^3$. Since we know the diameter of the spot, we can determine the thickness of the layer ablated: $\rho = \frac{m}{V} = \frac{m}{\pi r^2 t}$, so that:

$$t = \frac{m}{\pi r^2 \rho} = \frac{1.975\times10^{-10}\ \text{kg}}{\pi(5.00\times10^{-4}\ \text{m})^2(1000\ \text{kg/m}^3)} = 2.515\times10^{-7}\ \mu\text{m} = \underline{0.251\ \mu\text{m}}$$

(b) Yes, this thickness that the shape of the cornea can be very finely controlled, producing normal distant vision in more than 90% of patients.

CHAPTER 27: WAVE OPTICS

27.1 THE WAVE ASPECT OF LIGHT: INTERFERENCE

1. *Show that when light passes from air to water, its wavelength decreases to 0.750 times its original value.*

Solution Using the equation $\lambda_n = \dfrac{\lambda}{n}$, we can calculate the wavelength of light in water. The index of refraction for water is given in Table 25.1, so that $\lambda_n = \dfrac{\lambda}{n} = \dfrac{\lambda}{1.333} = \underline{0.750\,\lambda}$. The wavelength of light in water is 0.750 times the wavelength in air.

27.3 YOUNG'S DOUBLE SLIT EXPERIMENT

7. *Calculate the angle for the third-order maximum of 580-nm wavelength yellow light falling on double slits separated by 0.100 mm.*

Solution Using the equation $d \sin\theta = m\lambda \text{ for } m = 0,1,2,3,\ldots\ldots$ we can calculate the angle for $m = 3$, given the wavelength and the slit separation:

$$\theta = \sin^{-1}\left(\frac{m\lambda}{d}\right) = \sin^{-1}\left[\frac{(3)(5.80\times 10^{-7}\text{ m})}{1.00\times 10^{-4}\text{ m}}\right] = \underline{0.997^\circ}$$

13. *What is the highest-order maximum for 400-nm light falling on double slits separated by $25.0\ \mu\text{m}$?*

Solution Using the equation $d \sin\theta = m\lambda$, we notice that the highest order occurs when $\sin\theta = 1$, so the highest order is: $m = \dfrac{d}{\lambda} = \dfrac{2.50\times 10^{-5}\text{ m}}{4.00\times 10^{-7}\text{ m}} = 62.5$

Since *m* must be an integer, the highest order is then $\underline{m = 62.}$

19. *Using the result of the problem above, calculate the distance between fringes for 633-nm light falling on double slits separated by 0.0800 mm, located 3.00 m from a screen as in Figure 27.56.*

Solution From Exercise 25.18, we have an expression for the distance between the fringes , so that: $\Delta y = \frac{x\lambda}{d} = \frac{(3.00\text{ m})(6.33\times10^{-7}\text{ m})}{8.00\times10^{-5}\text{ m}} = 2.37\times10^{-2}\text{ m} = \underline{2.37\text{ cm}}$

27.4 MULTIPLE SLIT DIFFRACTION

25. *Calculate the wavelength of light that has its second-order maximum at* $45.0°$ *when falling on a diffraction grating that has 5000 lines per centimeter.*

Solution The second order maximum is constructive interference, so for diffraction gratings we use the equation $d\sin\theta = m\lambda \text{ for m} = 0,1,2,3,...$ where the second order maximum has $m = 2$. Next, we need to determine the slit separation by using the fact that there are 5000 lines per centimeter: $d = \frac{1}{5000\text{ slits/cm}} \times \frac{1\text{ m}}{100\text{ cm}} = 2.00\times10^{-6}\text{ m}$

So, since $\theta = 45.0°$, we can determine the wavelength of the light:

$$\lambda = \frac{d\sin\theta}{m} = \frac{(2.00\times10^{-6}\text{ m})(\sin 45.0°)}{2} = 7.07\times10^{-3}\text{ m} = \underline{707\text{ nm}}$$

34. *Show that a diffraction grating cannot produce a second-order maximum for a given wavelength of light unless the first-order maximum is at an angle less than* $30.0°$.

Solution The largest possible second order occurs when $\sin\theta_2 = 1$. Using the equation $d\sin\theta_m = m\lambda,$ we see that the value for the slit separation and wavelength are the same for the first and second order maximums, so that:

$d\sin\theta_1 = \lambda$ and $d\sin\theta_2 = 2\lambda$, so that: $\frac{\sin\theta_1}{\sin\theta_2} = \frac{1}{2}$

Now, since we know the maximum value for $\sin\theta_2$, we can solve for the maximum value for θ_1: $\theta_1 = \sin\left(\frac{1}{2}\sin\theta_2\right)^{-1}$ so that $\theta_{1,\max} = \sin\left(\frac{1}{2}\right)^{-1} = \underline{30.0°}$

37. *(a) Show that a 30,000-line-per-centimeter grating will not produce a maximum for visible light. (b) What is the longest wavelength for which it does produce a first-order maximum? (c) What is the greatest number of lines per centimeter a diffraction grating can have and produce a complete second-order spectrum for visible light?*

Solution (a) First we need to calculate the slit separation:

$$d=\frac{1\,\text{line}}{N}=\frac{1\,\text{line}}{30{,}000\,\text{lines/cm}}\times\frac{1\,\text{m}}{100\,\text{cm}}=3.333\times10^{-7}\ \text{m}=333.3\,\text{nm}.$$

Next, using the equation $d\sin\theta=m\lambda$, we see that the longest wavelength will be for $\sin\theta=1\,\text{and}\,\text{m}=1$, so in that case, $d=\lambda=333.3\,\text{nm}$, which is not visible.

(b) From part (a), we know that the longest wavelength is equal to the slit separation, or 333 nm.

(c) To get the largest number of lines per cm and still produce a complete spectrum, we want the smallest slit separation that allows the longest wavelength of visible light to produce a second order maximum, so $\lambda_{\max}=760\,\text{nm}$ (see Example 27.3). For there to be a second order spectrum, $m=2\,\text{and}\sin\theta=1,$ so

$$d=2\lambda_{\max}=2(760\ \text{nm})=1.52\times10^{-6}\ \text{m}$$

Now, using the technique in step (a), only in reverse:

$$N=\frac{1\,\text{line}}{d}=\frac{1\,\text{line}}{1.52\times10^{-6}\ \text{m}}\times\frac{1\,\text{m}}{100\,\text{cm}}=\underline{6.58\times10^{3}\ \text{lines/cm}}$$

41. ***Unreasonable Results*** *(a) What visible wavelength has its fourth-order maximum at an angle of* $25.0°$ *when projected on a 25,000-line-per-centimeter diffraction grating? (b) What is unreasonable about this result? (c) Which assumptions are unreasonable or inconsistent?*

Solution (a) For diffraction gratings, we use the equation $d\sin\theta=m\lambda,\,\text{for}\ m=0,1,2,3,4,...$ where the fourth order maximum has $m=4$. We first need to determine the slit separation by using the fact that there are 25,000 lines per centimeter:

$$d=\frac{1}{25{,}000\,\text{lines /cm}}\times\frac{1\,\text{m}}{100\,\text{cm}}=4.00\times10^{-7}\ \text{m}$$

So, since $\theta=25.0°$, we can determine the wavelength of the light:

$$\lambda=\frac{d\sin\theta}{m}=\frac{(4.00\times10^{-7}\ \text{m})(\sin25.0°)}{4}=4.226\times10^{-8}\ \text{m}=\underline{42.3\,\text{nm}}$$

(b) This wavelength is not in the visible spectrum.

(c) The number of slits in this diffraction grating is too large. Etching in integrated circuits can be done to a resolution of 50 nm, so slit separations of 400 nm are at the limit of what we can do today. This line spacing is too small to produce diffraction of light.

27.5 SINGLE SLIT DIFFRACTION

48. *Calculate the wavelength of light that produces its first minimum at an angle of* $36.9°$ *when falling on a single slit of width* $1.00\ \mu\text{m}$*.*

Solution Using the equation $D\sin\theta = m\lambda$, where D is the slit width, we can determine the wavelength for the first minimum,

$$\lambda = \frac{D\sin\theta}{m} = \frac{(1.00\times10^{-6}\ \text{m})(\sin 36.9°)}{1} = 6.004\times10^{-7}\ \text{m} = \underline{600\ \text{nm}}$$

54. *A double slit produces a diffraction pattern that is a combination of single and double slit interference. Find the ratio of the width of the slits to the separation between them, if the first minimum of the single slit pattern falls on the fifth maximum of the double slit pattern. (This will greatly reduce the intensity of the fifth maximum.)*

Solution The problem is asking us to find the ratio of D to d. For the single slit, using the equation $D\sin\theta = n\lambda$, we have $n = 1$. For the double slit using the equation $d\sin\theta = m\lambda$ (because we have a maximum), we have $m = 5$. Dividing the single slit equation by double slit equation, where the angle and wavelength are the same gives: $\frac{D}{d} = \frac{n}{m} = \frac{1}{5} \Rightarrow \underline{\frac{D}{d} = 0.200}$

So, the slit separation is five times the slit width.

55. ***Integrated Concepts*** *A water break at the entrance to a harbor consists of a rock barrier with a 50.0-m-wide opening. Ocean waves of 20.0-m wavelength approach the opening straight on. At what angle to the incident direction are the boats inside the harbor most protected against wave action?*

Solution We are looking for the first minimum for single slit diffraction because the 50.0 m wide opening acts as a single slit. Using the equation $D\sin\theta = m\lambda$, where $m=1$, we can determine the angle for first minimum:

$$\theta = \sin^{-1}\left(\frac{m\lambda}{D}\right) = \sin^{-1}\left[\frac{(1)(20.0\text{ m})}{50.0\text{ m}}\right] = 23.58° = \underline{23.6°}$$

Since the main peak for single slit diffraction is the main problem, a boat in the harbor at an angle greater than this first diffraction minimum will feel smaller waves. At the second minimum, the boat will not be affected by the waves at all:

$$\theta = \sin^{-1}\left(\frac{m\lambda}{D}\right) = \sin^{-1}\left[\frac{(2)(20.0\text{ m})}{50.0\text{ m}}\right] = 53.13° = \underline{53.1°}$$

27.6 LIMITS OF RESOLUTION: THE RAYLEIGH CRITERION

62. *The limit to the eye's acuity is actually related to diffraction by the pupil. (a) What is the angle between two just-resolvable points of light for a 3.00-mm-diameter pupil, assuming an average wavelength of 550 nm? (b) Take your result to be the practical limit for the eye. What is the greatest possible distance a car can be from you if you can resolve its two headlights, given they are 1.30 m apart? (c) What is the distance between two just-resolvable points held at an arm's length (0.800 m) from your eye? (d) How does your answer to (c) compare to details you normally observe in everyday circumstances?*

Solution (a) Using Rayleigh's Criterion, we can determine the angle (in radians) that is just resolvable: $\theta = 1.22\dfrac{\lambda}{D} = (1.22)\left(\dfrac{550\times10^{-9}\text{ m}}{3.00\times10^{-3}\text{ m}}\right) = 2.237\times10^{-4}\text{ rad} = \underline{2.24\times10^{-4}\text{ rad}}$

(b) The distance s between two objects, a distance r away, separated by an angle θ is $s = r\theta$, so $r = \dfrac{s}{\theta} = \dfrac{1.30\text{ m}}{2.237\times10^{-4}\text{ rad}} = 5.811\times10^{3}\text{ m} = \underline{5.81\text{ km}}$

(c) Using the same equation as in part (b):

$$s = r\theta = (0.800\text{ m})(2.237\times10^{-4}\text{ rad}) = 1.789\times10^{-4}\text{ m} = \underline{0.179\text{ mm}}$$

(d) Holding a ruler at arm's length, you can easily see the millimeter divisions; so you can resolve details 1.0 mm apart. Therefore, you probably can resolve details 0.2 mm apart at arm's length.

27.7 THIN FILM INTERFERENCE

73. *Find the minimum thickness of a soap bubble that appears red when illuminated by white light perpendicular to its surface. Take the wavelength to be 680 nm, and assume the same index of refraction as water.*

Solution The minimum thickness will occur when there is one phase change, so for light incident perpendicularly, constructive interference first occurs when $2t = \frac{\lambda_n}{2} = \frac{\lambda}{2n}$.

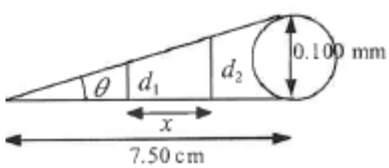
So, using the index of refraction for water from Table 25.1:

$$t = \frac{\lambda}{4n} = \frac{6.80 \times 10^{-7}\text{ m}}{(4)(1.33)} = 1.278 \times 10^{-7}\text{ m} = \underline{128\text{ nm}}$$

79. *Figure 27.34 shows two glass slides illuminated by pure-wavelength light incident perpendicularly. The top slide touches the bottom slide at one end and rests on a 0.100-mm-diameter hair at the other end, forming a wedge of air. (a) How far apart are the dark bands, if the slides are 7.50 cm long and 589-nm light is used? (b) Is there any difference if the slides are made from crown or flint glass? Explain.*

Solution

(a)



Two adjacent dark bands will have thickness differing by one wavelength, i.e.,

$$\lambda = d_2 - d_1, \text{ and } \tan\theta = \frac{\text{hair diameter}}{\text{slide length}} \text{ or } \theta = \tan^{-1}\left(\frac{1.00 \times 10^{-4}\text{ m}}{0.075\text{ m}}\right) = 0.076394°..$$

So, since $x\tan\theta = d_2 - d_1 = \lambda$, we see that

$$x = \frac{\lambda}{\tan\theta} = \frac{5.89 \times 10^{-7}\text{ m}}{\tan(0.076394°)} = 4.418 \times 10^{-4}\text{ m} = \underline{0.442\text{ mm}}$$

(b) The material makeup of the slides is irrelevant because it is the path difference in the air between the slides that gives rise to interference.

27.8 POLARIZATION

86. *If you have completely polarized light of intensity* $150\ \text{W/m}^2$*, what will its intensity be after passing through a polarizing filter with its axis at an* 89.0° *angle to the light's polarization direction?*

Solution Using the equation $I = I_0 \cos^2 \theta$, we can calculate the intensity:

$$I = I_0 \cos^2 \theta = (150\ \text{W/m}^2)\cos^2(89.0^\circ) = 4.57\times10^{-2}\ \text{W/m}^2 = \underline{45.7\ \text{m W/m}^2}$$

92. *What is Brewster's angle for light traveling in water that is reflected from crown glass?*

Solution Using the equation $\tan\theta_b = \frac{n_2}{n_1}$, where n_2 is for crown glass and n_1 is for water (see Table 25.1), Brewster's angle is $\theta_b = \tan^{-1}\left(\frac{n_2}{n_1}\right) = \tan^{-1}\left(\frac{1.52}{1.333}\right) = 48.75^\circ = \underline{48.8^\circ}$

At 48.8° (Brewster's angle) the reflected light is completely polarized.

CHAPTER 28: SPECIAL RELATIVITY

28.2 SIMULTANEITY AND TIME DILATION

1. *(a) What is γ if $v = 0.250c$? (b) If $v = 0.500c$?*

Solution (a) Using the definition of γ, where $v = 0.250c$:

$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} = \left[1 - \frac{(0.250c)^2}{c^2}\right]^{-1/2} = \underline{1.0328}$$

(b) Again using the definition of γ, now where $v = 0.500c$:

$$\gamma = \left[1 - \frac{(0.500c)^2}{c^2}\right]^{-1/2} = 1.1547 = \underline{1.15}$$

Note that γ is unitless, and the results are reported to three digits to show the difference from 1 in each case.

6. *A neutron lives 900 s when at rest relative to an observer. How fast is the neutron moving relative to an observer who measures its life span to be 2065 s?*

Solution Using $\Delta t = \gamma \Delta t_0$, where $\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2}$, we see that:

$$\gamma = \frac{\Delta t}{\Delta t_0} = \frac{2065\text{ s}}{900\text{ s}} = 2.2944 = \left(1 - \frac{v^2}{c^2}\right)^{-1/2}.$$

Squaring the equation gives $\gamma^2 = \left(\frac{c^2 - v^2}{c^2}\right)^{-1} = \frac{c^2}{c^2 - v^2}$

Cross-multiplying gives $c^2 - v^2 = \frac{c^2}{\gamma^2}$, and solving for speed finally gives:

$$v = \sqrt{c^2 - \frac{c^2}{\gamma^2}} = c\left(1 - \frac{1}{\gamma^2}\right)^{1/2} = c\left[1 - \frac{1}{(2.2944)^2}\right]^{1/2} = 0.90003c = \underline{0.900c}$$

11. ***Unreasonable Results*** *(a) Find the value of γ for the following situation. An Earth-bound observer measures 23.9 h to have passed while signals from a high-velocity space probe indicate that $24.0\,\text{h}$ have passed on board. (b) What is unreasonable about this result? (c) Which assumptions are unreasonable or inconsistent?*

Solution (a) Using the equation $\Delta t = \gamma \Delta t_0$, we can solve for γ:

$$\gamma = \frac{\Delta t}{\Delta t_0} = \frac{23.9\,\text{h}}{24.0\,\text{h}} = 0.9958 = \underline{0.996}$$

(b) γ cannot be less than 1.

(c) The earthbound observer must measure a longer time in the observer's rest frame than the ship bound observer. The assumption that time is longer in the moving ship is unreasonable.

28.3 LENGTH CONTRACTION

14. *(a) How far does the muon in Example 28.1 travel according to the Earth-bound observer? (b) How far does it travel as viewed by an observer moving with it? Base your calculation on its velocity relative to the Earth and the time it lives (proper time). (c) Verify that these two distances are related through length contraction $\gamma = 3.20$.*

Solution Using the values given in Example 28.1:

(a) $L_0 = v\Delta t = (0.950c)(4.87\times10^{-6}\ \text{s})$

$= (0.950c)(2.998\times10^{8}\ \text{m/s})(4.87\times10^{-6}\ \text{s}) = 1.386\times10^{3}\ \text{m} = \underline{1.39\ \text{km}}$

(b) $L_0 = v\Delta t = (0.950c)(2.998\times10^{8}\ \text{m/s})(1.52\times10^{-6}\ \text{s}) = 4.329\times10^{2}\ \text{m} = \underline{0.433\ \text{km}}$

(c) $L = \dfrac{L_0}{\gamma} = \dfrac{1.386\times10^{3}\ \text{m}}{3.20} = 4.33\times10^{2}\ \text{m} = \underline{0.433\ \text{km}}$

28.4 RELATIVISTIC ADDITION OF VELOCITIES

22. *If a spaceship is approaching the Earth at* $0.100c$ *and a message capsule is sent toward it at* $0.100c$ *relative to the Earth, what is the speed of the capsule relative to the ship?*

Solution Using the equation $u = \frac{v+u'}{1+(vu'/c^2)}$, we can add the relativistic velocities:

$$u = \frac{v+u'}{1+(vu'/c^2)} = \frac{0.100c + 0.100c}{1+\left[(0.100c)(0.100c)/c^2\right]} = \underline{0.198c}$$

28. *When a missile is shot from one spaceship towards another, it leaves the first at* $0.950c$ *and approaches the other at* $0.750c$. *What is the relative velocity of the two ships?*

Solution We are given: $u = 0.750c$ and $u' = 0.950c$. We want to find v, starting with the equation $u = \frac{v+u'}{1+(vu'/c^2)}$. First multiply both sides by the denominator:

$u + v\frac{uu'}{c^2} = v + u'$, then solving for v gives:

$$v = \frac{u'-u}{(uu'/c^2)-1} = \frac{0.950c - 0.750c}{\left[(0.750c)(0.950c)/c^2\right]-1} = \underline{-0.696c}$$

The velocity v is the speed measured by the second spaceship, so the minus sign indicates the ships are moving apart from each other (v is in the opposite direction as u).

34. *(a) All but the closest galaxies are receding from our own Milky Way Galaxy. If a galaxy* $12.0\times10^{9}\ \text{ly}$ *away is receding from us at* $0.900c$, *at what velocity relative to us must we send an exploratory probe to approach the other galaxy at* $0.990c$, *as measured from that galaxy? (b) How long will it take the probe to reach the other galaxy as measured from the Earth? You may assume that the velocity of the other galaxy remains constant. (c) How long will it then take for a radio signal to be beamed back? (All of this is possible in principle, but not practical.)*

Solution Note that all answers to this problem are reported to 5 significant figures, to distinguish the results.

(a) We are given $v = -0.900c$ and $u = 0.990c$. Starting with the equation

$u = \dfrac{v + u'}{1 + (vu'/c^2)}$, we now want to solve for u'. First multiply both sides by the denominator: $u + u'\dfrac{uv}{c^2} = v + u'$; then solving for the probe's speed gives:

$$u' = \frac{u - v}{1 - (uv/c^2)} = \frac{0.990c - (-0.900c)}{1 - [(0.990c)(-0.900c)/c^2]} = \underline{0.99947c}$$

(b) When the probe reaches the other galaxy, it will have traveled a distance (as seen on Earth) of $d = x_0 + vt$ because the galaxy is moving away from us. As seen from Earth, $u' = 0.9995c$, so $t = \dfrac{d}{u'} = \dfrac{x_0 + vt}{u'}$.

Now, $u't = x_0 + vt$ and $t = \dfrac{x_0}{u' - v} = \dfrac{12.0 \times 10^9 \text{ ly}}{0.99947c - 0.900c} \times \dfrac{(1\text{ y})c}{1\text{ ly}} = \underline{1.2064 \times 10^{11}\text{ y}}$

(c) The radio signal travels at the speed of light, so the return time t' is given by $t' = \dfrac{d}{c} = \dfrac{x_0 + vt}{c}$, assuming the signal is transmitted as soon as the probe reaches the other galaxy. Using the numbers, we can determine the time:

$$t' = \frac{1.20 \times 10^{10}\text{ ly} + (0.900\text{c})(1.2064 \times 10^{11}\text{ y})}{c} = \underline{1.2058 \times 10^{11}\text{ y}}$$

28.5 RELATIVISTIC MOMENTUM

39. *What is the velocity of an electron that has a momentum of $3.04 \times 10^{-21}\text{ kg}\cdot\text{m/s}$? Note that you must calculate the velocity to at least four digits to see the difference from c.*

Solution

Beginning with the equation $p = \gamma mu = \dfrac{mu}{\left[1 - \left(u^2/c^2\right)\right]^{1/2}}$ we can solve for the speed u.

First cross-multiply and square both sides, giving $1 - \dfrac{u^2}{c^2} = \dfrac{m^2}{p^2}u^2$

Then, solving for u^2 gives $u^2 = \dfrac{1}{\left(m^2/p^2\right) + \left(1/c^2\right)} = \dfrac{p^2}{m^2 + \left(p^2/c^2\right)}$

Finally, taking the square root gives $u = \dfrac{p}{\sqrt{m^2 + \left(p^2 / c^2\right)}}$.

Taking the values for the mass of the electron and the speed of light to five significant figures gives:

$$u = \frac{3.04\times10^{-21}\ \text{kg}\cdot\text{m/s}}{\left\{\left(9.1094\times10^{-31}\ \text{kg}\right)^2 + \left[\left(3.34\times10^{-21}\ \text{kg}\cdot\text{m/s}\right)/\left(2.9979\times10^{8}\ \text{kg}\right)^2\right]^2\right\}^{1/2}}$$
$$= \underline{2.988\times10^{8}\ \text{m/s}}$$

28.6 RELATIVISTIC ENERGY

48. *(a) Using data from Table 7.1, calculate the mass converted to energy by the fission of 1.00 kg of uranium. (b) What is the ratio of mass destroyed to the original mass,* $\Delta m / m$ *?*

Solution (a) From Table 7.1, the energy released from the nuclear fission of 1.00 kg of uranium is $\Delta E = 8.0\times10^{13}\ \text{J}$. So, $\Delta E_0 = E_{\text{released}} = \Delta mc^2$, we get

$$\Rightarrow \Delta m = \frac{\Delta E}{c^2} = \frac{\left(8.0\times10^{13}\ \text{J}\right)}{\left(3.00\times10^{8}\ \text{m/s}\right)^2} = 8.89\times10^{-4}\ \text{kg} = \underline{0.89\ \text{g}}$$

(b) To calculate the ratio, simply divide by the original mass:

$$\frac{\Delta m}{m} = \frac{\left(8.89\times10^{-4}\ \text{kg}\right)}{\left(1.00\ \text{kg}\right)^2} = 8.89\times10^{-4} = \underline{8.9\times10^{-4}}$$

52. *A* π *-meson is a particle that decays into a muon and a massless particle. The* π *-meson has a rest mass energy of 139.6 MeV, and the muon has a rest mass energy of 105.7 MeV. Suppose the* π *-meson is at rest and all of the missing mass goes into the muon's kinetic energy. How fast will the muon move?*

Solution Using the equation $\text{KE}_{\text{rel}} = \Delta mc^2$, we can determine the kinetic energy of the muon by determining the missing mass: $\text{KE}_{\text{rel}} = \Delta mc^2 = (m_\pi - m_\mu)c^2 = (\gamma - 1)m_\mu c^2$

Solving for γ will give us a way of calculating the speed of the muon. From the equation above, we see that:

$$\gamma = \frac{m_\pi - m_\mu}{m_\mu} + 1 = \frac{m_\pi - m_\mu - m_\mu}{m_\mu} = \frac{m_\pi}{m_\mu} = \frac{139.6\ \text{MeV}}{105.7\ \text{MeV}} = 1.32072.$$

Now, use $\gamma = \left[1 - \left(v^2/c^2\right)\right]^{-1/2}$ or $1 - \frac{v^2}{c^2} = \frac{1}{\gamma^2}$, so

$$v = c\sqrt{1 - \frac{1}{\gamma^2}} = c\sqrt{1 - \frac{1}{(1.32072)^2}} = \underline{0.6532c}$$

58. *Find the kinetic energy in MeV of a neutron with a measured life span of 2065 s, given its rest energy is 939.6 MeV, and rest life span is 900s.*

Solution From Exercise 28.6, we know that $\gamma = 2.2944$, so we can determine the kinetic energy of the neutron: $\text{KE}_{\text{rel}} = (\gamma - 1)mc^2 = (2.2944 - 1)(939.6\ \text{MeV}) = \underline{1216\ \text{MeV}}$.

64. *(a) Calculate the energy released by the destruction of 1.00 kg of mass. (b) How many kilograms could be lifted to a 10.0 km height by this amount of energy?*

Solution (a) Using the equation $E = mc^2$, we can calculate the rest mass energy of a 1.00 kg mass. This rest mass energy is the energy released by the destruction of that amount of mass:

$$E_{\text{released}} = mc^2 = (1.00\ \text{kg})(2.998 \times 10^8\ \text{m/s}) = 8.988 \times 10^{16}\ \text{J} = \underline{8.99 \times 10^{16}\ \text{J}}$$

(b) Using the equation $\text{PE} = mgh$, we can determine how much mass can be raised to a height of 10.0 km: $m = \frac{\text{PE}}{gh} = \frac{8.99 \times 10^{16}\ \text{J}}{(9.80\ \text{m/s}^2)(10.0 \times 10^3\ \text{m})} = \underline{9.17 \times 10^{11}\ \text{kg}}$

CHAPTER 29: INTRODUCTION TO QUANTUM PHYSICS

29.1 QUANTIZATION OF ENERGY

1. *A LiBr molecule oscillates with a frequency of* $1.7\times10^{13}\ \text{Hz}$. *(a) What is the difference in energy in eV between allowed oscillator states? (b) What is the approximate value of* n *for a state having an energy of 1.0 eV?*

Solution (a) $\Delta E = hf = \left(6.63\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(1.7\times10^{13}\ \text{s}^{-1}\right) = 1.127\times10^{-20}\ \text{J}$

so that $\left(1.127\times10^{-20}\ \text{J}\right)\left(\dfrac{1\ \text{eV}}{1.60\times10^{-19}\ \text{J}}\right) = 7.04\times10^{-2}\ \text{eV} = \underline{7.0\times10^{-2}\ \text{eV}}$

(b) Using the equation $E = nhf$, we can solve for n:

$$n = \frac{E}{hf} - \frac{1}{2} = \frac{(1.0\ \text{eV})\left(1.60\times10^{-19}\ \text{J/eV}\right)}{\left(6.63\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(1.7\times10^{13}\ \text{s}^{-1}\right)} - \frac{1}{2} = 13.7 = \underline{14}$$

29.2 THE PHOTOELECTRIC EFFECT

7. *Calculate the binding energy in eV of electrons in aluminum, if the longest-wavelength photon that can eject them is 304 nm.*

Solution The longest wavelength corresponds to the shortest frequency, or the smallest energy. Therefore, the smallest energy is when the kinetic energy is zero. From the equation $\text{KE} = hf - \text{BE} = 0$, we can calculate the binding energy (writing the frequency in terms of the wavelength):

$$\text{BE} = hf = \frac{hc}{\lambda} \Rightarrow$$

$$\text{BE} = \frac{hc}{\lambda} = \frac{\left(6.63\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(3.00\times10^{8}\ \text{m/s}\right)}{\left(3.04\times10^{-7}\ \text{m}\right)}$$

$$= 6.543\times10^{-19}\ \text{J}\times\left(\frac{1.000\ \text{eV}}{1.60\times10^{-19}\ \text{J}}\right) = \underline{4.09\ \text{eV}}$$

13. *Find the wavelength of photons that eject 0.100-eV electrons from potassium, given that the binding energy is 2.24 eV. Are these photons visible?*

Solution Using the equations $\text{KE} = hf - \text{BE}$ and $c = \lambda f$ we see that $hf = \frac{hc}{\lambda} = \text{BE} + \text{KE}$ so that we can calculate the wavelength of the photons in terms of energies:

$$\lambda = \frac{hc}{\text{BE} + \text{KE}} = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{2.24\ \text{eV} + 0.100\ \text{eV}} \times \left(\frac{1.000\ \text{eV}}{1.60\times10^{-19}\ \text{J}}\right)$$
$$= 5.313\times10^{-7}\ \text{m} = \underline{531\ \text{nm}}.$$

Yes, these photons are visible.

19. ***Unreasonable Results*** *(a) What is the binding energy of electrons to a material from which 4.00-eV electrons are ejected by 400-nm EM radiation? (b) What is unreasonable about this result? (c) Which assumptions are unreasonable or inconsistent?*

Solution (a) We want to use the equation $\text{KE} = hf - \text{BE}$ to determine the binding energy, so we first need to determine an expression of hf. Using $E = hf$, we know:

$$hf = \frac{hc}{\lambda} = \frac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(2.998\times10^{8}\ \text{m/s})}{4.00\times10^{-7}\ \text{m}}$$
$$= (4.966\times10^{-19}\ \text{J})\left(\frac{1\ \text{eV}}{(1.602\times10^{-19}\ \text{J})}\right) = 3.100\ \text{eV}$$

and since $\text{KE} = hf - \text{BE}$: $\text{BE} = hf - \text{KE} = 3.100\ \text{eV} - 4.00\ \text{eV} = \underline{-0.90\ \text{eV}}$

(b) The binding energy is too large for the given photon energy.

(c) The electron's kinetic energy is too large for the given photon energy; it cannot be greater than the photon energy.

29.3 PHOTON ENERGIES AND THE ELECTROMAGNETIC SPECTRUM

21. *(a) Find the energy in joules and eV of photons in radio waves from an FM station that has a 90.0-MHz broadcast frequency. (b) What does this imply about the number of photons per second that the radio station must broadcast?*

Solution (a) Using the equation $E = hf$ we can determine the energy of photons:

$$E = hf = (6.63\times10^{-34}\ \text{J/s})(9.00\times10^{8}\ \text{s}^{-1}) = \underline{5.97\times10^{-26}\ \text{J}}$$
$$= 5.97\times10^{-26}\ \text{J}\left(\frac{1\ \text{eV}}{1.60\times10^{-19}\ \text{J}}\right) = \underline{3.73\times10^{-7}\ \text{eV}}$$

(b) This implies that a tremendous number of photons must be broadcast per second. In order to have a broadcast power of, say 50.0 kW, it would take

$$\frac{5.00\times10^{4}\ \text{J/s}}{5.97\times10^{-26}\ \text{J/photon}} = \underline{8.38\times10^{29}\ \text{photon/sec}}$$

24. *Do the unit conversions necessary to show that* $hc = 1240\ \text{eV}\cdot\text{nm}$*, as stated in the text.*

Solution Using the conversion for joules to electron volts and meters to nanometers gives:

$$hc = (6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})\left(\frac{10^{9}\ \text{nm}}{1\ \text{m}}\right)\left(\frac{1.00\ \text{eV}}{1.60\times10^{-19}\ \text{J}}\right) = \underline{1240\ \text{eV}\cdot\text{nm}}$$

33. *(a) If the power output of a 650-kHz radio station is 50.0 kW, how many photons per second are produced? (b) If the radio waves are broadcast uniformly in all directions, find the number of photons per second per square meter at a distance of 100 km. Assume no reflection from the ground or absorption by the air.*

Solution (a) We can first calculate the energy of each photon:

$$E_{\gamma} = hf = (6.63\times10^{-34}\ \text{Js})(6.50\times10^{5}\ \text{s}^{-1}) = \underline{4.31\times10^{-28}\ \text{J}}$$

Then using the fact that the broadcasting power is 50.0 kW, we can calculate the number of photons per second:

$$N = \frac{5.00\times10^{4}\ \text{J/s}}{4.31\times10^{-28}\ \text{J/photon}} = 1.16\times10^{32}\ \text{photon/s} = \underline{1.16\times10^{32}\ \text{photon/s}}$$

(b) To calculate the flux of photons, we assume that the broadcast is uniform in all directions, so the area is the surface area of a sphere giving:

$$\Phi_N = \frac{N}{4\pi r^2} = \frac{1.16\times10^{32}\text{ photons/s}}{4\pi\left(1.00\times10^5\text{ m}\right)^2} = \underline{9.23\times10^{20}\text{ photons/s}\cdot\text{m}^2}$$

29.4 PHOTON MOMENTUM

40. *(a) What is the wavelength of a photon that has a momentum of* $5.00\times10^{-29}\text{ kg}\cdot\text{m/s}$*? (b) Find its energy in eV.*

Solution (a) Using the equation $p=\dfrac{h}{\lambda}$, we can solve for the wavelength of the photon:

$$\lambda = \frac{h}{p} = \frac{6.63\times10^{-34}\text{ J.s}}{5.00\times10^{-29}\text{ kg}\cdot\text{m/s}} = 1.326\times10^{-5}\text{ m} = \underline{13.3\ \mu\text{m}}$$

(b) Using the equation $p=\dfrac{E}{c}$, we can solve for the energy and then convert the units to electron volts:

$$E = pc = \left(5.00\times10^{-29}\text{ kg}\cdot\text{m/s}\right)\left(3.00\times10^{8}\text{ m/s}\right)$$
$$= 1.50\times10^{-20}\text{ J}\times\left(\frac{1\text{ eV}}{1.60\times10^{-19}\text{ J}}\right) = \underline{9.38\times10^{-2}\text{ eV}}$$

46. *Take the ratio of relativistic rest energy,* $E=\gamma mc^2$*, to relativistic momentum,* $p=\gamma mu$*, and show that in the limit that mass approaches zero, you find* $E/p=c$.

Solution Beginning with the two equations $E=\gamma mc^2$ and, $p=\gamma mu$ gives $\dfrac{E}{p}=\dfrac{\gamma mc^2}{\gamma mu}=\dfrac{c^2}{u}$

As the mass of the particle approaches zero, its velocity u will approach c so that the ratio of energy to momentum approaches $\lim_{m\to0}\dfrac{E}{p}=\dfrac{c^2}{c}=c$, which is consistent with the equation $p=\dfrac{E}{c}$ for photons.

29.6 THE WAVE NATURE OF MATTER

54. *Experiments are performed with ultracold neutrons having velocities as small as 1.00 m/s. (a) What is the wavelength of such a neutron? (b) What is its kinetic energy in eV?*

Solution (a) Using the equations $p = \frac{h}{\lambda}$ and $p = mv$ we can calculate the wavelength of the neutron:

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{(1.675\times10^{-27}\ \text{kg})(1.00\ \text{m/s})} = 3.956\times10^{-7}\ \text{m} = \underline{396\ \text{nm}}$$

(b) Using the equation $\text{KE} = \frac{1}{2}mv^2$ we can calculate the kinetic energy of the neutron:

$$\text{KE} = \frac{1}{2}mv^2 = \frac{1}{2}\times(1.675\times10^{-27}\ \text{kg})(1.00\ \text{m/s})^2$$
$$= 8.375\times10^{-28}\ \text{J}\times\left(\frac{1\ \text{eV}}{1.602\times10^{-19}\ \text{J}}\right) = \underline{5.23\times10^{-9}\ \text{eV}}$$

29.7 PROBABILITY: THE HEISENBERG UNCERTAINTY PRINCIPLE

66. *A relatively long-lived excited state of an atom has a lifetime of 3.00 ms. What is the minimum uncertainty in its energy?*

Solution Using the equation $\Delta E \Delta t = \frac{h}{4\pi}$, we can determine the minimum uncertainty for its energy:

$$\Delta E \geq \frac{h}{4\pi\Delta t} = \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{4\pi(3.00\times10^{-3}\ \text{s})} = 1.759\times10^{-32}\ \text{J}\times\left(\frac{1\ \text{eV}}{1.60\times10^{-19}\ \text{J.}}\right) = \underline{1.10\times10^{-13}\ \text{eV}}$$

29.8 THE PARTICLE-WAVE DUALITY REVIEWED

72. ***Integrated Concepts*** *The 54.0-eV electron in Example 29.7 has a 0.167-nm wavelength. If such electrons are passed through a double slit and have their first maximum at an angle of* $25.0°$*, what is the slit separation* d*?*

Solution Using the equation $d\sin\theta = m\lambda, m = 0,1,2....$, we can solve for the slit separation when $m = 1$ for the first order maximum: $d = \dfrac{m\lambda}{\sin\theta} = \dfrac{(1)(0.167\ nm)}{\sin 25.0°} = \underline{0.395\ \text{nm}}$

78. ***Integrated Concepts*** *(a) What is γ for a proton having an energy of 1.00 TeV, produced by the Fermilab accelerator? (b) Find its momentum. (c) What is the proton's wavelength?*

Solution (a) Using the equation $E = \gamma mc^2$, we can find γ for 1.00 TeV proton:

$$\gamma = \frac{E}{mc^2} = \frac{(1.00\times10^{12}\ \text{eV})(1.60\times10^{-19}\ \text{J/eV})}{(1.6726\times10^{-27}\ \text{kg})(3.00\times10^{8}\ \text{m/s}^2)^2} = 1.063\times10^3 = \underline{1.06\times10^3}$$

(b) $p = \gamma mc = \dfrac{E}{c} = \dfrac{(1.00\times10^{12}\ \text{eV})(1.60\times10^{-19}\ \text{J/eV})}{3.00\times10^{8}\ \text{m/s}} = \underline{5.33\times10^{-16}\ \text{kg}\cdot\text{m/s}}$

(c) Using the equation $p = \dfrac{h}{\lambda}$, we can calculate the proton's wavelength:

$$\lambda = \frac{h}{p} = \frac{6.63\times10^{-34}\ \text{kg}\cdot\text{m/s}}{5.33\times10^{-16}\ \text{kg}\cdot\text{m/s}} = \underline{1.24\times10^{-18}\ \text{m}}$$

83. ***Integrated Concepts*** *One problem with x rays is that they are not sensed. Calculate the temperature increase of a researcher exposed in a few seconds to a nearly fatal accidental dose of x rays under the following conditions. The energy of the x-ray photons is 200 keV, and 4.00×10^{13} of them are absorbed per kilogram of tissue, the specific heat of which is $0.830\ \text{kcal/kg}\cdot°\text{C}$. (Note that medical diagnostic x-ray machines* cannot *produce an intensity this great.)*

Solution First, we know the amount of heat absorbed by 1.00 kg of tissue is equal to the number of photons times the energy each one carry, so:

$$Q = NE_\gamma = (4.00\times10^{13})(2.00\times10^{5}\ \text{eV})\left(\frac{1.60\times10^{-19}\ \text{J}}{1\ \text{eV}}\right) = \underline{1.28\ \text{J}}$$

Next, using the equation $Q = mc\Delta t$, we can determine how much 1.00 kg tissue is heated: $\Delta t = \dfrac{Q}{mc} = \dfrac{1.282\ \text{J}}{(1.00\ \text{kg})(0.830\ \text{kcal/kg}\cdot°\text{C})(4186\ \text{J/kcal})} = \underline{3.69\times10^{-4}\ °\text{C}}$

CHAPTER 30: ATOMIC PHYSICS

30.1 DISCOVERY OF THE ATOM

1. *Using the given charge-to-mass ratios for electrons and protons, and knowing the magnitudes of their charges are equal, what is the ratio of the proton's mass to the electron's? (Note that since the charge-to-mass ratios are given to only three-digit accuracy, your answer may differ from the accepted ratio in the fourth digit.)*

Solution We can calculate the ratio of the masses by taking the ratio of the charge to mass ratios given: $\frac{q}{m_e} = 1.76\times10^{11}\ \text{C/kg}$ and $\frac{q}{m_p} = 9.57\times10^{7}\ \text{C/kg}$, so that

$$\frac{m_p}{m_e} = \frac{q/m_e}{q/m_p} = \frac{1.76\times10^{11}\ \text{C/kg}}{9.57\times10^{7}\ \text{C/kg}} = 1839 = 1.84\times10^{3}\ .$$

The actual mass ratio is: $\frac{m_p}{m_e} = \frac{1.6726\times10^{-27}\ \text{kg}}{9.1094\times10^{-31}\ \text{kg}} = 1836 = 1.84\times10^{3}$, so to three digits, the mass ratio is correct.

30.3 BOHR'S THEORY OF THE HYDROGEN ATOM

12. *A hydrogen atom in an excited state can be ionized with less energy than when it is in its ground state. What is n for a hydrogen atom if 0.850 eV of energy can ionize it?*

Solution Using $E_n = \frac{-13.6\ \text{eV}}{n^2}$, we can determine the value for n, given the ionization energy:

$$n = \sqrt{\frac{-13.6\ \text{eV}}{E_n}} = \left(\frac{-13.6\ \text{eV}}{-0.85\ \text{eV}}\right)^{1/2} = 4.0 = \underline{4}$$

(Remember that n must be an integer.)

18. *(a) Which line in the Balmer series is the first one in the UV part of the spectrum? (b) How many Balmer series lines are in the visible part of the spectrum? (c) How many are in the UV?*

Solution (a) We know that the UV range is from $\lambda = 10\ \text{nm}$ to approximately $\lambda = 380\ \text{nm}$. Using the equation $\frac{1}{\lambda} = R\left(\frac{1}{n_\text{f}^2} - \frac{1}{n_i^2}\right)$, where $n_\text{f} = 2$ for the Balmer series, we can solve for n_i. Finding a common denominator gives $\frac{1}{\lambda R} = \frac{n_\text{i}^2 - n_\text{f}^2}{n_\text{i}^2 n_\text{f}^2}$, so that $n_\text{i}^2 n_\text{f}^2 = \lambda R(n_\text{i}^2 - n_\text{f}^2)$, or $n_\text{i} = n_\text{f}\sqrt{\frac{\lambda R}{\lambda R - n_\text{f}^2}}$. The first line will be for the lowest energy photon, and therefore the largest wavelength, so setting $\lambda = 380\ \text{nm}$ gives $n_\text{i} = 2\sqrt{\frac{(3.80\times10^{-7}\ \text{m})(1.097\times10^{7}\ \text{m}^{-1})}{(3.80\times10^{-7}\ \text{m})(1.097\times10^{7}\ \text{m}^{-1}) - 4}} = 9.94 \Rightarrow n_\text{i} = \underline{10}$ will be the first.

(b) Setting $\lambda = 760\ \text{nm}$ allows us to calculate the smallest value for n_i in the visible range: $n_\text{i} = 2\sqrt{\frac{(7.60\times10^{-7}\ \text{m})(1.097\times10^{7}\ \text{m}^{-1})}{(7.60\times10^{-7}\ \text{m})(1.097\times10^{7}\ \text{m}^{-1}) - 4}} = 2.77 \Rightarrow n_\text{i} = 3$ so $n_\text{i} = 3\ \text{to}\ 9$ are visible, or 7 lines are in the visible range.

(c) The smallest λ in the Balmer series would be for $n_\text{i} = \infty$, which corresponds to a value of:

$$\frac{1}{\lambda} = R\left(\frac{1}{n_\text{f}^2} - \frac{1}{n_\text{i}^2}\right) = \frac{R}{n_\text{f}^2} \Rightarrow \lambda = \frac{n_\text{f}^2}{R} = \frac{4}{1.097\times10^{7}\ \text{m}^{-1}} = 3.65\times10^{-7}\ \text{m} = 365\ \text{nm},$$

which is in the ultraviolet. Therefore, there are an infinite number of Balmer lines in the ultraviolet. All lines from $n_\text{i} = 10\ \text{to}\ \infty$ fall in the ultraviolet part of the spectrum.

23. *1. Verify Equations* $r_n = \frac{n^2}{Z} a_\text{B}$ *and* $a_B = \frac{h^2}{4\pi^2 m_e k q_e^2} = 0.529\times10^{-10}\ \text{m}$ *using the approach stated in the text. That is, equate the Coulomb and centripetal forces and then insert an expression for velocity from the condition for angular momentum quantization.*

Solution Using $F_{\text{coulomb}} = F_{\text{centripetal}} \Rightarrow \dfrac{kZq_e^2}{r_n^2} = \dfrac{m_e v^2}{r_n}$, so that $r_n = \dfrac{kZq_e^2}{m_e v^2} = \dfrac{kZq_e^2}{m_e}\dfrac{1}{v^2}$.

Since $m_e v r_n = n\dfrac{h}{2\pi}$, we can substitute for the velocity giving: $r_n = \dfrac{kZq_e^2}{m_e} \cdot \dfrac{4\pi^2 m_e^2 r_n^2}{n^2 h^2}$

so that $r_n = \dfrac{n^2}{Z}\dfrac{h^2}{4\pi^2 m_e k q_e^2} = \dfrac{n^2}{Z} a_{\text{B}}$, where $a_{\text{B}} = \dfrac{h^2}{4\pi^2 m_e k q_e^2}$.

30.4 X RAYS: ATOMIC ORIGINS AND APPLICATIONS

26. *A color television tube also generates some x rays when its electron beam strikes the screen. What is the shortest wavelength of these x rays, if a 30.0-kV potential is used to accelerate the electrons? (Note that TVs have shielding to prevent these x rays from exposing viewers.)*

Solution Using the equations $E = qV$ and $E = \dfrac{hc}{\lambda}$ gives $E = qV = \dfrac{hc}{\lambda}$, which allows us to calculate the wavelength:

$$\lambda = \frac{hc}{qV} = \frac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(2.998\times10^{8}\ \text{m/s})}{(1.602\times10^{-19}\ \text{C})(3.00\times10^{4}\ \text{V})} = \underline{4.13\times10^{-11}\ \text{m}}$$

30.5 APPLICATIONS OF ATOMIC EXCITATIONS AND DE-EXCITATIONS

33. *(a) What energy photons can pump chromium atoms in a ruby laser from the ground state to its second and third excited states? (b) What are the wavelengths of these photons? Verify that they are in the visible part of the spectrum.*

Solution (a) From Figure 30.64, we see that it would take 2.3 eV photons to pump chromium atoms into the second excited state. Similarly, it would take 3.0 eV photons to pump chromium atoms into the third excited state.

(b) $\lambda_2 = \frac{hc}{E} = \frac{1.24\times10^{-6}\ \text{eV}\cdot\text{m}}{2.3\ \text{eV}} = 5.39\times10^{-7}\ \text{m} = \underline{5.4\times10^2\ \text{nm}}$, which is yellow-green.

$$\lambda_2 = \frac{hc}{E} = \frac{1.24\times10^{-6}\ \text{eV}\cdot\text{m}}{3.0\ \text{eV}} = 4.13\times10^{-7}\ \text{m} = \underline{4.1\times10^2\ \text{nm}}$$, which is blue-violet.

30.8 QUANTUM NUMBERS AND RULES

40. *(a) What is the magnitude of the angular momentum for an $l = 1$ electron? (b) Calculate the magnitude of the electron's spin angular momentum. (c) What is the ratio of these angular momenta?*

Solution (a) Using the equation $L = \sqrt{\ell(\ell+1)}\frac{h}{2\pi}$, we can calculate the angular momentum of an $\ell = 1$ electron: $L = \sqrt{\ell(\ell+1)}\frac{h}{2\pi} = \sqrt{1(2)}\left(\frac{6.626\times10^{-34}\ \text{J}\cdot\text{s}}{2\pi}\right) = \underline{1.49\times10^{-34}\ \text{J}\cdot\text{s}}$

(b) Using the equation $S = \sqrt{s(s+1)}\frac{h}{2\pi}$, we can determine the electron's spin angular momentum, since $s = \frac{1}{2}$:

$$S = \sqrt{s(s+1)}\frac{h}{2\pi} = \sqrt{\frac{1}{2}\left(\frac{3}{2}\right)}\frac{6.626\times10^{-34}\ \text{J}\cdot\text{s}}{2\pi} = \underline{9.13\times10^{-35}\ \text{J}\cdot\text{s}}$$

(c) $\frac{L}{S} = \frac{\sqrt{\ell(\ell+1)}\,h/2\pi}{\sqrt{s(s+1)}\,h/2\pi} = \frac{\sqrt{2}}{\sqrt{3/4}} = \underline{1.63}$

30.9 THE PAULI EXCLUSION PRINCIPLE

55. ***Integrated Concepts*** *Calculate the velocity of a star moving relative to the earth if you observe a wavelength of 91.0 nm for ionized hydrogen capturing an electron directly into the lowest orbital (that is, a $n_\text{i} = \infty$ to $n_\text{f} = 1$, or a Lyman series transition).*

Solution We will use the equation $\Delta E = E_\text{f} - E_\text{i}$ to determine the speed of the star, since we are given the observed wavelength. We first need the source wavelength:

$$\Delta E = E_\text{f} - E_\text{i} = \frac{hc}{\lambda_\text{s}} = \left(-\frac{Z^2}{n_\text{f}^2}E_0\right) - \left(-\frac{Z^2}{n_\text{i}^2}E_0\right) = 0 - \left[-\frac{1^2}{1^2}(13.6\,\text{eV})\right] = 13.6\,\text{eV},$$

so that $\lambda_\text{s} = \frac{hc}{\Delta E} = \frac{1.24\times10^3\ \text{eV}\cdot\text{nm}}{13.6\,\text{eV}} = 91.2\,\text{nm}$. Therefore, using $\lambda_\text{obs} = \lambda_\text{s}\sqrt{\frac{1+v/c}{1-v/c}}$, we have $\frac{1+v/c}{1-v/c} = \frac{\lambda_\text{obs}^2}{\lambda_\text{s}^2}$, so that $1+\frac{v}{c} = \frac{\lambda_\text{obs}^2}{\lambda_\text{s}^2}\left(1-\frac{v}{c}\right)$ and thus,

$$\frac{v}{c} = \frac{\lambda_\text{obs}^2/\lambda_\text{s}^2 - 1}{\lambda_\text{obs}^2/\lambda_\text{s}^2 + 1} = \frac{(91.0\,\text{nm}/91.2\,\text{nm})^2 - 1}{(91.0\,\text{nm}/91.2\,\text{nm})^2 + 1} = -2.195\times10^{-3}.$$

So, $v = (-2.195\times10^{-3})(2.998\times10^8\ \text{m/s}) = -6.58\times10^5\ \text{m/s}$. Since v is negative, the star is moving toward the earth at a speed of $\underline{6.58\times10^5\ \text{m/s}}$.

59. ***Integrated Concepts*** *Find the value of l, the orbital angular momentum quantum number, for the moon around the earth. The extremely large value obtained implies that it is impossible to tell the difference between adjacent quantized orbits for macroscopic objects.*

Solution From the definition of velocity, $v = \frac{d}{t}$, we can get an expression for the velocity in terms of the period of rotation of the moon: $v = \frac{2\pi R}{T}$. Then, from $L = I\omega$ for a point object we get the angular momentum: $L = I\omega = mR^2\omega = mR^2\frac{v}{R} = mRv$. Substituting for the velocity and setting equal to $L = \sqrt{\ell(\ell+1)}\frac{h}{2\pi}$ gives:

$L = mvR = \frac{2\pi\, mR^2}{T} = \sqrt{\ell(\ell+1)}\ \frac{h}{2\pi}$. Since l is large : $\frac{2\pi\, mR^2}{T} \approx \frac{\ell h}{2\pi}$, so

$$\ell = \frac{4\pi^2\, mR^2}{Th} = \frac{4\pi^2\,(7.35\times10^{22}\ \text{kg})(3.84\times10^8\ \text{m})^2}{(2.36\times10^6\ \text{s})(6.63\times10^{-34}\ \text{J.s})} = \underline{2.73\times10^{68}}\ .$$

66. ***Integrated Concepts*** *A pulsar is a rapidly spinning remnant of a supernova. It rotates on its axis, sweeping hydrogen along with it so that hydrogen on one side moves toward us as fast as 50.0 km/s, while that on the other side moves away as fast as 50.0 km/s. This means that the EM radiation we receive will be Doppler shifted over a range of* $\pm 50.0\ \text{km/s}$*. What range of wavelengths will we observe for the 91.20-nm line in the Lyman series of hydrogen? (Such line broadening is observed and actually provides part of the evidence for rapid rotation.)*

Solution We will use the Doppler shift equation to determine the observed wavelengths for the Doppler shifted hydrogen line. First, for the hydrogen moving away from us, we use $u = +50.0\ \text{km/s}$, so that:

$$\lambda_{\text{obs}} = (91.20\ \text{nm})\sqrt{\frac{1+\left(5.00\times10^{4}\ \text{m/s}/2.998\times10^{8}\ \text{m/s}\right)}{1-\left(5.00\times10^{4}\ \text{m/s}/2.998\times10^{8}\ \text{m/s}\right)}} = 91.22\ \text{nm}$$

Then, for the hydrogen moving towards us, we use $u = -50.0\ \text{km/s}$, so that:

$$\lambda_{\text{obs}} = (91.20\ \text{nm})\sqrt{\frac{1-\left(5.00\times10^{4}\ \text{m/s}/2.998\times10^{8}\ \text{m/s}\right)}{1+\left(5.00\times10^{4}\ \text{m/s}/2.998\times10^{8}\ \text{m/s}\right)}} = 91.18\ \text{nm}$$

The range of wavelengths is from $\underline{91.18\ \text{nm to } 91.22\ \text{nm}}$.

CHAPTER 31: RADIOACTIVITY AND NUCLEAR PHYSICS

31.2 RADIATION DETECTION AND DETECTORS

1. *The energy of 30.0* eV *is required to ionize a molecule of the gas inside a Geiger tube, thereby producing an ion pair. Suppose a particle of ionizing radiation deposits 0.500 MeV of energy in this Geiger tube. What maximum number of ion pairs can it create?*

Solution To calculate the number of pairs created, simply divide the total energy by energy needed per pair: $\#\text{pairs} = \dfrac{(0.500\text{ MeV})(1.00\times 10^6\text{ eV/MeV})}{30.0\text{ eV/pair}} = \underline{1.67\times 10^4\text{ pairs}}$. This is the maximum number of ion pairs because it assumes all the energy goes to creating ion pairs and that there are no energy losses.

31.3 SUBSTRUCTURE OF THE NUCLEUS

9. *(a) Calculate the radius of* ^{58}Ni*, one of the most tightly bound stable nuclei. (b) What is the ratio of the radius of* ^{58}Ni *to that of* ^{258}Ha*, one of the largest nuclei ever made? Note that the radius of the largest nucleus is still much smaller than the size of an atom.*

Solution (a) Using the equation $r = r_0 A^{1/3}$ we can approximate the radius of ^{58}Ni:

$$r_{\text{Ni}} = r_0 A_{\text{Ni}}^{1/3} = (1.2\times 10^{-15}\text{ m})(58)^{1/3} = 4.6\times 10^{-15}\text{ m} = \underline{4.6\text{ fm}}$$

(b) Again using this equation this time we can approximate the radius of ^{258}Ha:

$$r_{\text{Ha}} = (1.2\times 10^{-15}\text{ m})(258)^{1/3} = 7.6\times 10^{-15}\text{ m} = \underline{7.6\text{ fm}}.$$

Finally, taking the ratio of Ni to Ha gives: $\dfrac{r_{\mathrm{Ni}}}{r_{\mathrm{Ha}}}=\dfrac{4.645\times10^{-15}\ \mathrm{m}}{7.639\times10^{-15}\ \mathrm{m}}=\underline{0.61\ \mathrm{to}\ 1}$

15. *What is the ratio of the velocity of a 5.00-MeV β ray to that of an α particle with the same kinetic energy? This should confirm that βs travel much faster than αs even when relativity is taken into consideration. (See also Exercise 31.11.)*

Solution We know that the kinetic energy for a relativistic particle is given by the equation $\mathrm{KE}_{\mathrm{rel}}=(\gamma-1)mc^2$, and that since $\gamma=\left(1-\dfrac{v^2}{c^2}\right)^{-1/2}$, we can get an expression for the speed $\dfrac{v^2}{c^2}=\left(1-\dfrac{1}{\gamma^2}\right)$, or $v=c\sqrt{1-\dfrac{1}{\gamma^2}}$

For the β particle: $\mathrm{KE}=5.00\ \mathrm{MeV}=(\gamma-1)(0.511\ \mathrm{MeV})$, so that $(\gamma-1)=9.785$ or $\gamma=10.785$. Thus, the velocity for the β particle is:

$$v_\beta=c\sqrt{1-\frac{1}{\gamma^2}}=(2.998\times10^8\ \mathrm{m/s})\sqrt{1-\frac{1}{(10.785)^2}}=2.985\times10^8\ \mathrm{m/s}$$

For the α particle: $\mathrm{KE}=5.00\ \mathrm{MeV}=(\gamma-1)(4.0026\ \mathrm{u})\left(\dfrac{931.5\ \mathrm{MeV/u}}{c^2}\right)c^2$ so that $\gamma=1.00134$. Thus, the velocity for the α particle is:

$v_\alpha=(2.998\times10^8\ \mathrm{m/s})\sqrt{1-\dfrac{1}{(1.00134)^2}}=1.551\times10^7\ \mathrm{m/s}$. Finally, the ratio of the velocities is given by: $\dfrac{v_\beta}{v_\alpha}=\dfrac{2.985\times10^8\ \mathrm{m/s}}{1.55\times10^7\ \mathrm{m/s}}=\underline{19.3\ \mathrm{to}\ 1}$.

In other words, when the β and α particles have the same kinetic energy , the β particle is approximately 19 times faster than the α particle.

31.4 NUCLEAR DECAY AND CONSERVATION LAWS

22. *Electron capture by* ^{106}In.

Solution Referring to the electron capture equation, ${}^{A}_{Z}X_{N} + e^{-} \rightarrow {}^{A}_{Z-1}Y_{N+1} + v_{e}$, we need to calculate the values of Z and N. From the periodic table we know that indium has $Z = 49$ and the element with $Z = 48$ is cadmium. Using the equation $A = N + Z$ we know that $N = A\text{-}Z = 106\text{-}49 = 57$ for indium and $N = 58$ for cadmium. Putting this all together gives $\underline{{}^{106}_{49}\text{In}_{57} + e^{-} \rightarrow {}^{106}_{48}\text{Cd}_{58} + v_{e}}$

28. α *decay producing* ^{208}Pb. *The parent nuclide is in the decay series produced by* ^{232}Th, *the only naturally occurring isotope of thorium.*

Solution Since we know that ^{208}Pb is the product of an α decay, ${}^{A}_{Z}X_{N} \rightarrow {}^{A-4}_{Z-2}Y_{N-2} + {}^{4}_{2}\text{He}_{2}$ tells us that $A - 4 = 208$, and since $Z - 2 = 82$ from the periodic table, we then know that $N - 2 = 208 - 82 = 126.$ So for the parent nucleus we have $A = 212, Z = 84$ and $N = 128$. Therefore from the periodic table the parent nucleus is ^{212}Po and the decay is $\underline{{}^{212}_{84}\text{Po}_{128} \rightarrow {}^{208}_{82}\text{Pb}_{126} + {}^{4}_{2}\text{He}_{2}}$

34. *A rare decay mode has been observed in which* ^{222}Ra *emits a* ^{14}C *nucleus. (a) The decay equation is* $^{222}\text{Ra} \rightarrow {}^{A}\text{X} + {}^{14}\text{C}$. *Identify the nuclide* ^{A}X. *(b) Find the energy emitted in the decay. The mass of* ^{222}Ra *is 222.015353 u.*

Solution (a) The decay is ${}^{222}_{88}\text{Ra}_{134} \rightarrow {}^{A}_{Z}\text{X}_{N} + {}^{14}_{6}\text{C}_{8}$, so we know that:

$A = 222 - 14 = 208; Z = 88 - 6 = 82$ and $N = A - Z = 208 - 82 = 126,$ so from the periodic table the element is lead and $\underline{\text{X} = {}^{208}_{82}\text{Pb}_{126}}$

(b) $\Delta m = m\left({}^{222}_{88}\text{Ra}_{134}\right) - m\left({}^{208}_{82}\text{Pb}_{126}\right) - m\left({}^{14}_{6}\text{C}_{8}\right)$

$= 222.015353\text{ u} - 207.976627\text{ u} - 14.003241\text{ u} = 3.5485\times 10^{-2}\text{ u}$

$$E = \Delta mc^2 = \left(3.5485\times 10^{-2}\text{ u}\right)\left(\frac{931.5\text{ MeV}/c^2}{\text{u}}\right)c^2 = \underline{33.05\text{ MeV}}$$

40. *(a) Write the complete β^+ decay equation for ${}^{11}\text{C}$. (b) Calculate the energy released in the decay. The masses of ${}^{11}\text{C}$ and ${}^{11}\text{B}$ are 11.011433 and 11.009305 u, respectively.*

Solution (a) Using ${}^{A}_{Z}X_{N} \rightarrow {}^{A}_{Z-1}Y_{N+1} + \beta^+ + v_e$ and the periodic table we can get the complete decay equation: $\underline{{}^{11}_{6}\text{C}_5 \rightarrow {}^{11}_{5}\text{B}_6 + \beta^+ + v_e}$

(b) To calculate the energy emitted we first need to calculate the change in mass. The change in mass is the mass of the parent minus the mass of the daughter and the positron it created. The mass given for the parent and the daughter, however, are given for the neutral atoms. So the carbon has one additional electron than the boron and we must subtract an additional mass of the electron to get the correct change in mass.

$\Delta m = m\left({}^{11}\text{C}\right) - \left[m\left({}^{11}\text{B}\right) + 2m_e\right]$

$= 11.011433\text{ u} - \left[11.009305\text{ u} + 2\left(0.00054858\text{ u}\right)\right] = 1.031\times 10^{-3}\text{ u}$

$$E = \Delta mc^2 = \left(1.031\times 10^{-3}\text{ u}\right)\left(\frac{931.5\text{ MeV}/c^2}{\text{u}}\right)c^2 = \underline{0.9602\text{ MeV}}$$

31.5 HALF-LIFE AND ACTIVITY

46. *(a) Calculate the activity R in curies of 1.00 g of ${}^{226}\text{Ra}$. (b) Discuss why your answer is not exactly 1.00 Ci, given that the curie was originally supposed to be exactly the activity of a gram of radium.*

Solution (a) First we must determine the number of atoms for radium. We use the molar mass of 226 g/mol to get: $N = (1.00\ \text{g})\left(\dfrac{\text{mol}}{226\ \text{g}}\right)\dfrac{6.022\times10^{23}\ \text{atoms}}{\text{mol}} = 2.6646\times10^{21}\ \text{atoms}$

Then using the equation $R = \dfrac{0.693N}{t_{1/2}}$, where we know the half life of ^{226}Ra is $1.6\times10^{3}\ \text{y}$,

$$R = \frac{(0.693)(2.6646\times10^{21})}{1.6\times10^{3}\ \text{y}}\times\left(\frac{1\ \text{y}}{3.156\times10^{7}\ \text{s}}\right) = 3.66\times10^{10}\ \text{Bq}$$

$$\left(\frac{\text{Ci}}{3.70\times10^{16}\ \text{Bq}}\right) = \underline{0.988\ \text{Ci}}$$

(b) The half life of ^{226}Ra is more accurately known than it was when the Ci unit was established.

52. ^{50}V *has one of the longest known radioactive half-lives. In a difficult experiment, a researcher found that the activity of 1.00 kg of* ^{50}V *is 1.75 Bq. What is the half-life in years?*

Solution Using the equation $R = \dfrac{0.693N}{t_{1/2}}$, we can write the activity in terms of the half-life, the molar mass, M, and the mass of the sample, m:

$$R = \frac{0.693N}{t_{1/2}} = \frac{(0.693)\left[(6.02\times10^{23}\ \text{atoms/mol})/M\right]m}{t_{1/2}}$$

From the periodic table, $M = 50.94\ \text{g/mol}$, so

$$t_{1/2} = \frac{(0.693)(6.02\times10^{23}\ \text{atoms/mol})(1000\ \text{g})}{(50.94\ \text{g/mol})(1.75\ \text{Bq})}$$

$$= 4.681\times10^{24}\ \text{s}\left(\frac{1\ \text{y}}{3.156\times10^{7}\ \text{s}}\right) = \underline{1.48\times10^{17}\ \text{y}}$$

58. *The β^- particles emitted in the decay of $^3\mathrm{H}$ (tritium) interact with matter to create light in a glow-in-the-dark exit sign. At the time of manufacture, such a sign contains 15.0 Ci of $^3\mathrm{H}$. (a) What is the mass of the tritium? (b) What is its activity 5.00 y after manufacture?*

Solution Using the equation $R = \dfrac{0.693N}{t_{1/2}}$, we can write the activity in terms of the half-life, the atomic mass, M and the mass of the sample m:

(a) $R = \dfrac{0.693N}{t_{1/2}} = \dfrac{(0.693)(m/M)}{t_{1/2}}$. The atomic mass of tritium (from Appendix A) is

$M = 3.016050\,\mathrm{u}\left(\dfrac{1.6605\times10^{-27}\ \mathrm{kg}}{1\,\mathrm{u}}\right) = 5.0082\times10^{-27}\ \mathrm{kg/atom}$, and the half-life is 12.33 y (from Appendix B), so we can determine the original mass of tritium:

$m = \dfrac{Rt_{1/2}M}{0.693}$ or

$$m = \frac{(15.0\,\mathrm{Ci})(12.33\ \mathrm{y})(5.0082\times10^{-27}\ \mathrm{kg})M}{0.693}\left(\frac{3.70\times10^{10}\ \mathrm{Bq}}{\mathrm{Ci}}\right)\left(\frac{3.156\times10^{7}\ \mathrm{s}}{\mathrm{y}}\right)$$

$$= 1.56\times10^{-6}\ \mathrm{kg} = \underline{1.56\ \mathrm{mg}}$$

(b) $R = R_0 e^{-\lambda t} = R_0 \exp\left(-\dfrac{0.693}{t_{1/2}}t\right) = (15.0\,\mathrm{Ci})\exp\left(-\dfrac{0.693(5.00\ \mathrm{y})}{12.33\ \mathrm{y}}\right) = \underline{11.3\,\mathrm{Ci}}$

31.6 BINDING ENERGY

71. *$^{209}\mathrm{Bi}$ is the heaviest stable nuclide, and its BE/A is low compared with medium-mass nuclides. Calculate BE/A, the binding energy per nucleon, for $^{209}\mathrm{Bi}$ and compare it with the approximate value obtained from the graph in Figure 31.27.*

Solution Dividing $\mathrm{BE} = \{[Zm(^1\mathrm{H}) + Nm_n] - m(^AX)\}c^2$ by A gives the binding energy per nucleon: $\frac{\text{BE}}{A}=\frac{\left[Zm\left({}^{1}\text{H}\right)+Nm_n-m\left({}^{209}_{83}\text{Bi}_{126}\right)\right]c^2}{A}$.

We know that $Z=83$ (from the periodic table), $N=A-Z=126$ and the mass of the ^{209}Bi nuclide is 208.908374 u (from Appendix A) so that:

$$\frac{\text{BE}}{A}=\frac{[83(1.007825\text{ u})+126(1.008665\text{ u})-208.980374\text{ u}]}{209}\left(\frac{931.5\text{ MeV}/c^2}{\text{u}}\right)c^2$$
$$=\underline{7.848\text{ MeV/nucleon}}$$

This binding energy per nucleon is approximately the value given in the graph.

76. ***Unreasonable Results*** *A particle physicist discovers a neutral particle with a mass of 2.02733 u that he assumes is two neutrons bound together. (a) Find the binding energy. (b) What is unreasonable about this result? (c) What assumptions are unreasonable or inconsistent?*

Solution (a) $\text{BE}=[2m_n-m(\text{particle})]c^2=[2(1.008665\text{ u})-2.02733\text{ u}]\left(\frac{931.5\text{ MeV}/c^2}{\text{u}}\right)c^2$
$$=\underline{-9.315\text{ MeV}}$$

(b) The binding energy cannot be negative; the nucleons would not stay together.

(c) The particle cannot be made from two neutrons.

31.7 TUNNELING

78. ***Integrated Concepts*** *A 2.00-T magnetic field is applied perpendicular to the path of charged particles in a bubble chamber. What is the radius of curvature of the path of a 10 MeV proton in this field? Neglect any slowing along its path.*

Solution Using the equation $r=\frac{mv}{qB}$, we can determine the radius of a moving charge in a magnetic field. First, we need to determine the velocity of the proton. Since the energy of the proton (10.0 MeV) is substantially less than the rest mass energy of the proton (938 MeV), we know the velocity is non-relativistic and that $E = \frac{1}{2}mv^2$.

Therefore,

$$v = \left(\frac{2E}{m}\right)^{1/2} = \left(\frac{2\cdot 10.0\ \text{MeV}}{938.27\ \text{MeV}/\text{c}^2}\right)^{1/2} = (0.1460)(2.998\times 10^7\ \text{m/s}).\ \text{So,}$$

$$r = \frac{mv}{qB} = \frac{(1.6726\times 10^{-27}\ \text{kg})(4.377\times 10^7\ \text{m/s})}{(1.602\times 10^{-19}\ \text{C})(2.00\ \text{T})} 0.228\ \text{m} = \underline{22.8\ \text{cm}}$$

CHAPTER 32: MEDICAL APPLICATIONS OF NUCLEAR PHYSICS

32.1 MEDICAL IMAGING AND DIAGNOSTICS

1. *A neutron generator uses an α source, such as radium, to bombard beryllium, inducing the reaction ${}^{4}\text{He} + {}^{9}\text{Be} \rightarrow {}^{12}\text{C} + n$. Such neutron sources are called RaBe sources, or PuBe sources if they use plutonium to get the α s. Calculate the energy output of the reaction in MeV.c*

Solution Using $E = \Delta mc^2$, we can determine the energy output of the reaction by calculating the change in mass of the constituents in the reaction, where the masses are found either in Appendix A or Table 31.2:

$$\begin{aligned} E &= (m_\text{i} - m_\text{f})c^2 \\ &= (4.002603 + 9.02182 - 12.000000 - 1.008665)(931.5\ \text{MeV}) = \underline{5.701\,\text{MeV}} \end{aligned}$$

6. *The activities of ${}^{131}\text{I}$ and ${}^{123}\text{I}$ used in thyroid scans are given in Table 32.1 to be 50 and $70\ \mu\text{Ci}$, respectively. Find and compare the masses of ${}^{131}\text{I}$ and ${}^{123}\text{I}$ in such scans, given their respective half-lives are 8.04 d and 13.2 h. The masses are so small that the radioiodine is usually mixed with stable iodine as a carrier to ensure normal chemistry and distribution in the body.*

Solution Beginning with the equation $R = \dfrac{0.693N}{t_{1/2}} = \dfrac{(0.693)(m/M)N_\text{A}}{t_{1/2}}$ we can solve for the mass of the iodine isotopes, where the atomic masses and the half lives are given in the appendices:

$$m_{131} = \frac{RMt_{1/2}}{0.693 N_A} = \frac{(5.0\times10^{-5}\text{ Ci})(3.70\times10^{10}\text{ Bq/Ci})(130.91\text{ g/mol})(8.040\text{ d})(86400\text{ s/d})}{(0.693)(6.02\times10^{23})}$$
$$= \underline{4.0\times10^{-10}\text{ g}}$$

$$m_{123} = \frac{RMt_{1/2}}{0.693 N_A} = \frac{(7.0\times10^{-5}\text{ Ci})(3.70\times10^{10}\text{ Bq/Ci})(122.91\text{ g/mol})(13.2\text{ h})(3600\text{ s/h})}{(0.693)(6.02\times10^{23})}$$
$$= \underline{3.6\times10^{-11}\text{ g}}$$

32.2 BIOLOGICAL EFFECTS OF IONIZING RADIATION

10. *How many Gy of exposure is needed to give a cancerous tumor a dose of 40 Sv if it is exposed to α activity?*

Solution Using the equation $\text{Sv} = \text{Gy}\times\text{RBE}$ and Table 32.2, we know that $\text{RBE} = 20$ for whole body exposure, so $\text{Gy} = \frac{\text{Sv}}{\text{RBE}} = \frac{40\text{ Sv}}{20} = \underline{2\text{ Gy}}$

32.3 THERAPEUTIC USES OF IONIZING RADIATION

21. *Large amounts of ^{65}Zn are produced in copper exposed to accelerator beams. While machining contaminated copper, a physicist ingests $50.0\ \mu\text{Ci of }^{65}\text{Zn}$. Each ^{65}Zn decay emits an average γ-ray energy of 0.550 MeV, 40.0% of which is absorbed in the scientist's 75.0-kg body. What dose in mSv is caused by this in one day?*

Solution First, we need to determine the number of decays per day:

$$\text{decays/day} = (5.00\times10^{-5}\text{ Ci})(3.70\times10^{10}\text{ Bq/Ci})(8.64\times10^{4}\text{ s/d}) = 1.598\times10^{11}\text{ /d}$$

Next, we can calculate the energy because each decay emits an average of 0.550 MeV of energy:

$$E/day = \left(\frac{1.598\times10^{11}\text{ decays}}{\text{d}}\right)(0.400)\left(\frac{0.550\text{ MeV}}{\text{decay}}\right)\left(\frac{1.602\times10^{-13}\text{ J}}{\text{MeV}}\right)$$
$$= 5.633\times10^{-3}\text{ J/d}$$

Then, dividing by the mass of tissue gives the dose:

Dose in rad/d $= \left(\dfrac{5.633\times10^{-3}\ \text{J/d}}{75.0\ \text{kg}}\right)\dfrac{1\ \text{rad}}{0.0100\ \text{J/kg}} = 7.51\times10^{-3}\ \text{rad/d}$

Finally, from Table 32.2, we see that the RBE is 1 for γ radiation, so:

$$\text{rem/d} = \text{rad}\times\text{RBE} = \left(7.51\times10^{-3}\ \text{rad/d}\right)\times(1) = 7.51\times10^{-3}\ \text{rem/d}\frac{\text{mSv}}{0.1\ \text{rem}}$$

$$= \underline{7.51\times10^{-4}\ \text{mSv/d}}$$

This dose is approximately $2700\ \text{mrem/y}$, which is larger than background radiation sources, but smaller than doses given for cancer treatments.

32.5 FUSION

30. *The energy produced by the fusion of a 1.00-kg mixture of deuterium and tritium was found in Example Calculating Energy and Power from Fusion. Approximately how many kilograms would be required to supply the annual energy use in the United States?*

Solution From Table 7.6, we know $E = 1.05\times10^{20}\ \text{J}$ and from Example 32.2, we know that a 1.00 kg mixture of deuterium and tritium releases $3.37\times10^{14}\ \text{J}$ of energy, so:

$$M = \left(1.05\times10^{20}\ \text{J}\right)\left(\frac{1.00\ \text{kg}}{3.37\times10^{14}\ \text{J}}\right) = \underline{3.12\times10^{5}\ \text{kg}}$$

35. *The power output of the Sun is $4\times10^{26}\ \text{W}$. (a) If 90% of this is supplied by the proton-proton cycle, how many protons are consumed per second? (b) How many neutrinos per second should there be per square meter at the Earth from this process? This huge number is indicative of how rarely a neutrino interacts, since large detectors observe very few per day.*

Solution (a) Four protons are needed for each cycle to occur. The energy released by a proton-proton cycle is $26.7\ \text{MeV}$, so that

$$\#\ \text{protons/s} = (0.90)\left(4\times10^{26}\ \text{J/s}\right)\left(\frac{4\ \text{protons}}{26.7\ \text{MeV}}\right)\left(\frac{1\ \text{MeV}}{1.602\times10^{-13}\ \text{J}}\right)$$

$$= \underline{3\times10^{38}\ \text{protons/s}}$$

(b) For each cycle, two neutrinos are created and four protons are destroyed. To determine the number of neutrinos at Earth, we need to determine the number of neutrinos leaving the Sun and divide that by the surface area of a sphere with radius from the Sun to Earth:

$$\frac{\#}{\text{area}}=\frac{\#}{4\pi R^2}=\frac{(2v_e/4\,\text{protons})(3.37\times10^{38}\ \text{protons/s})}{4\pi(1.50\times10^{11}\ \text{m})^2}=\underline{6\times10^{14}\ \text{neutrinos/m}^2\cdot\text{s}}$$

32.6 FISSION

45. *(a) Calculate the energy released in the neutron-induced fission reaction* $n+{}^{239}\text{Pu}\rightarrow{}^{96}\text{Sr}+{}^{140}\text{Ba}+4n$, *given* $m({}^{96}\text{Sr})=95.921750\ \text{u}$ *and* $m({}^{140}\text{Ba})=139.910581\ \text{u}$. *(b) Confirm that the total number of nucleons and total charge are conserved in this reaction.*

Solution (a) To calculate the energy released, we use $E=\Delta mc^2$ to calculate the difference in energy before and after the reaction:

$$\begin{aligned}E&=\left(m_n+m\left({}^{239}\text{Pu}\right)-m\left({}^{96}\text{Sr}\right)-m\left({}^{140}\text{Ba}\right)-4m_n\right)c^2\\&=\left(m\left({}^{239}\text{Pu}\right)-m\left({}^{96}\text{Sr}\right)-m\left({}^{140}\text{Ba}\right)-3m_n\right)c^2\\&=[239.052157-95.921750-139.910581-(3)(1.008665)](931.5\ \text{MeV})\\&=\underline{180.6\ \text{MeV}}\end{aligned}$$

(b) Writing the equation in full form gives ${}^{1}_{0}n_1+{}^{239}_{94}\text{Pu}_{145}\rightarrow{}^{96}_{38}\text{Sr}_{84}+4{}^{1}_{0}n_1$ so we can determine the total number of nucleons before and after the reaction and the total charge before and after the reaction:

$$\begin{aligned}A_\text{i}&=1+239=240=96+140+4=A_\text{f};\\Z_\text{i}&=0+94=94=56+38+4(0)=Z_\text{f}.\end{aligned}$$

Therefore, both the total number of nucleons and the total charge are conserved.

32.7 NUCLEAR WEAPONS

51. *Find the mass converted into energy by a 12.0-kT bomb.*

Solution Using $E=mc^2$, we can calculate the mass converted into energy for a 12.0 kT bomb:

$$m=\frac{E}{c^2}=\frac{(12.0\ \text{kT})(4.2\times10^{12}\ \text{J/kT})}{(3.00\times10^8\ \text{m/s})^2}=5.60\times10^{-4}\ \text{kg}=\underline{0.56\ \text{g}}$$

57. *Assume one-fourth of the yield of a typical 320-kT strategic bomb comes from fission reactions averaging 200 MeV and the remainder from fusion reactions averaging 20 MeV. (a) Calculate the number of fissions and the approximate mass of uranium and plutonium fissioned, taking the average atomic mass to be 238. (b) Find the number of fusions and calculate the approximate mass of fusion fuel, assuming an average total atomic mass of the two nuclei in each reaction to be 5. (c) Considering the masses found, does it seem reasonable that some missiles could carry 10 warheads? Discuss, noting that the nuclear fuel is only a part of the mass of a warhead.*

Solution (a) Given that for fission reactions, the energy produced is 200 MeV per fission, we can convert the $1/4$ of 320 kT yield into the number of fissions:

$$\#\text{ of fissions} = \frac{(1/4)(320\text{ kT})(4.2\times10^{12}\text{ J/kT})}{(200\text{ MeV/fission})(1.60\times10^{-13}\text{ J/MeV})} = \underline{1.1\times10^{25}\text{ fissions}}$$

Then, $m = (1.1\times10^{25}\text{ nuclei})\left(\frac{1\text{ mol}}{6.022\times10^{23}\text{ nuclei}}\right)(238\text{ g/mol}) = 4.35\times10^{3}\text{ g} = \underline{4.3\text{ kg}}$

(b) Similarly, given that for fusion reactions, the energy produced is 20 MeV per fusion, we convert the $3/4$ of 320 kT yield into the number of fusions:

$$\#\text{ of fusions} = \frac{(3/4)(320\text{ kT})(4.2\times10^{12}\text{ J/kT})}{(200\text{ MeV/fission})(1.60\times10^{-13}\text{ J/MeV})} = \underline{3.2\times10^{26}\text{ fusions}}$$

Then:

$$m = (3.2\times10^{26}\text{ fusions})\left(\frac{1\text{ mol}}{6.022\times10^{23}\text{ nuclei}}\right)(5\text{ g LiD fuel/mol}) = 2.66\times10^{3}\text{ g} = \underline{2.7\text{ kg}}$$

(c) The nuclear fuel totals only 6 kg, so it is quite reasonable that some missiles carry 10 overheads. The mass of the fuel would only be 60 kg and therefore the mass of the 10 warheads, weighing about 10 times the nuclear fuel, would be only 1500 lbs. If the fuel for the missiles weighs 5 times the total weight of the warheads, the missile would weigh about 9000 lbs or 4.5 tons. This is not an unreasonable weight for a missile.

CHAPTER 33: PARTICLE PHYSICS

33.2 THE FOUR BASIC FORCES

4. *(a) Find the ratio of the strengths of the weak and electromagnetic forces under ordinary circumstances. (b) What does that ratio become under circumstances in which the forces are unified?*

Solution (a) From Table 33.1, we know that the ratio of the weak force to the electromagnetic force is $\frac{\text{Weak}}{\text{Electromagnetic}} = \frac{10^{-13}}{10^{-2}} = \underline{10^{-11}}$. In other words, the weak force is 11 orders of magnitude weaker than the electromagnetic force.

(b) When the forces are unified, the idea is that the four forces are just different manifestations of the same force, so under circumstances in which the forces are unified, the ratio becomes <u>1 to 1</u>. (See Section 33.6.)

33.3 ACCELERATORS CREATE MATTER FROM ENERGY

7. *Suppose a W^- created in a bubble chamber lives for 5.00×10^{-25} s. What distance does it move in this time if it is traveling at 0.900c? Since this distance is too short to make a track, the presence of the W^- must be inferred from its decay products. Note that the time is longer than the given W^- lifetime, which can be due to the statistical nature of decay or time dilation.*

Solution Using the definition of velocity, we can determine the distance traveled by the W^- in a bubble chamber:

$$d = vt = (0.900)(3.00\times10^{8}\text{ m/s})(5.00\times10^{-25}\text{ s}) = 1.35\times10^{-16}\text{ m} = \underline{0.135\text{ fm}}$$

33.4 PARTICLES, PATTERNS, AND CONSERVATION LAWS

13. *The π^0 is its own antiparticle and decays in the following manner: $\pi^0 \rightarrow \gamma + \gamma$. What is the energy of each γ ray if the π^0 is at rest when it decays?*

Solution If the π^0 is at rest when it decays, its total energy is just $E = mc^2$. Since its initial momentum is zero, each γ ray will have equal but opposite momentum i.e. $p_i = 0 = p_f$, so that $p_{y_1} + p_{y_2} = 0$, or $p_{y_1} = -p_{y_2}$. Since a γ ray is a photon: $E_\gamma = |p_\gamma|c.$ Therefore, since the momenta are equal in magnitude the energies of the γ rays are equal: $E_1 = E_2$. Then, by conservation of energy, the initial energy of the π^0 equals twice the energy of one of the γ rays: $m_{\pi^0}c^2 = 2E.$ Finally, from Table 33.2, we can determine the rest mass energy of the π^0, and the energy of each γ ray is:

$$E = \frac{m_{\pi^\circ}c^2}{2} = \frac{\left(135\ \text{MeV}/c^2\right)c^2}{2} = \underline{67.5\ \text{MeV}}$$

19. *(a) What is the uncertainty in the energy released in the decay of a π^0 due to its short lifetime? (b) What fraction of the decay energy is this, noting that the decay mode is $\pi^0 \rightarrow \gamma + \gamma$ (so that all the π^0 mass is destroyed)?*

Solution (a) Using $\Delta E \Delta t \approx \dfrac{h}{4\pi}$, we can calculate the uncertainty in the energy, given the lifetime of the π^0 from Table 33.2:

$$\Delta E = \frac{h}{4\pi\Delta t} = \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{4\pi(8.4\times10^{-17}\ \text{s})} = 6.28\times10^{-19}\ \text{J}\times\frac{1\ \text{eV}}{1.60\times10^{-19}\ \text{J}} = \underline{3.9\ \text{eV}}$$

(b) The fraction of the decay energy is determined by dividing this uncertainty in the energy by the rest mass energy of the π^0 found in Table 33.2:

$$\frac{\Delta E}{m_{\pi^0}c^2} = \frac{3.9256\text{ eV}}{\left(135.0\times10^6\text{ eV}/c^2\right)c^2} = \underline{2.9\times10^{-8}}.$$

So the uncertainty is approximately 2.9×10^{-6} percent of the rest mass energy.

33.5 QUARKS: IS THAT ALL THERE IS?

25. *Repeat the previous problem for the decay mode* $\Omega^- \to \Lambda^0 + K^-$.

Solution (a) From Table 33.4, we know the quark composition of each of the particles involved in this decay: $\Omega^-(sss) \to \Lambda^0(uds) + K^-(\bar{u}s)$. Then, to determine the change in strangeness, we need to subtract the initial from the final strangeness, remembering that a strange quark has a strangeness of -1:

$$\Delta S = S_{\text{f}} - S_{\text{i}} = \left[-1+(-1)\right]-(-3) = \underline{+1}$$

(b) Using Table 33.3, we know that $B_{\text{i}} = \left(\frac{1}{3}+\frac{1}{3}+\frac{1}{3}\right) = 1$,

$B_{\text{f}} = \left(\frac{1}{3}+\frac{1}{3}+\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{3}\right) = 1$, so the baryon number is indeed conserved. Again, using Table 33.3, the charge is:

$Q_{\text{i}} = \left(-\frac{1}{3}-\frac{1}{3}-\frac{1}{3}\right)q_e = -q_e$, and $Q_{\text{f}} = \left(+\frac{2}{3}-\frac{1}{3}-\frac{1}{3}\right)q_e + \left(-\frac{2}{3}-\frac{1}{3}\right)q_e = -q_e$, so charge is indeed conserved. This decay does not involve any electrons or neutrinos, so all lepton numbers are zero before and after, and the lepton numbers are unaffected by the decay.

(c) Using Table 33.4, we can write the equation in terms of its constituent quarks: $\underline{(sss) \to (uds) + \left(\bar{u}s\right)}$ or $s \to u + \bar{u} + d$. Since there is a change in quark flavor, the weak nuclear force is responsible for the decay.

31. *(a) Is the decay* $\Sigma^- \to n + \pi^-$ *possible considering the appropriate conservation laws? State why or why not. (b) Write the decay in terms of the quark constituents of the*

particles.

Solution (a) From Table 33.4, we know the quark composition of each of the particles involved in the decay: $\Sigma^-(dds) \rightarrow n(udd) + \pi^-(\bar{u}d)$. The charge is conserved at -1. The baryon number is conserved at B=1. All lepton numbers are conserved at zero, and finally the mass initially is larger than the final mass: $m_{\Sigma^-} > (m_n + m_{\pi^-})$, so, yes, this decay is possible by the conservation laws.

(b) Using Table 33.4, we can write the equation in terms of its constituent quarks: $\underline{dds \rightarrow udd + \bar{u}d}$ or $s \rightarrow u + \bar{u} + d$

37. *(a) How much energy would be released if the proton did decay via the conjectured reaction $p \rightarrow \pi^0 + e^+$? (b) Given that the π^0 decays to two γ s and that the e^+ will find an electron to annihilate, what total energy is ultimately produced in proton decay? (c) Why is this energy greater than the proton's total mass (converted to energy)?*

Solution (a) The energy released from the reaction is determined by the change in the rest mass energies: $\Delta E = (mc^2)_\text{i} - \Sigma(mc^2)_\text{f} = (m_p - m_{\pi^0} - m_{e^+})c^2$

Using Table 33.2, we can then determine this difference in rest mass energies:

$$\Delta E = (938.3\ \text{MeV}/c^2 - 135.0\ \text{MeV}/c^2 - 0.511\ \text{MeV}/c^2)c^2 = \underline{802.8\ \text{MeV}} = 803\ \text{MeV}$$

(b) The two γ rays will carry a total energy of the rest mass energy of the π^0:

$$\pi^0 \rightarrow 2\gamma \Rightarrow \Delta E_{\pi^0} = m_{\pi^0}c^2 = 135.0\ \text{MeV}$$

The positron/electron annihilation will give off the rest mass energies of the positron and the electron:

$$e^- + e^+ \rightarrow 2\gamma \Rightarrow \Delta E_{e^+} = 2m_e c^2 = 2(0.511\ \text{MeV}) = 1.022\ \text{MeV}$$

So, the total energy would be the sum of all these energies:

$$\Delta E_\text{tot} = \Delta E + \Delta E_{\pi^0} + \Delta E_{e^+} = \underline{938.8\ \text{MeV}}$$

(c) Because the total energy includes the annihilation energy of an extra electron. So the full reaction should be $p + e \to \left(\pi^0 + e^+\right) + e \to 4\gamma$.

33.6 GUTS: THE UNIFICATION OF FORCES

43. ***Integrated Concepts*** *The intensity of cosmic ray radiation decreases rapidly with increasing energy, but there are occasionally extremely energetic cosmic rays that create a shower of radiation from all the particles they create by striking a nucleus in the atmosphere as seen in the figure given below. Suppose a cosmic ray particle having an energy of* $10^{10}\ \text{GeV}$ *converts its energy into particles with masses averaging* $200\ \text{MeV}/c^2$. *(a) How many particles are created? (b) If the particles rain down on a* 1.00-km^2 *area, how many particles are there per square meter?*

Solution (a) To determine the number of particles created, divide the cosmic ray particle energy by the average energy of each particle created:

$$\#\text{ of particles created} = \frac{\text{cosmic ray energy}}{\text{energy/particle created}} = \frac{10^{10}\ \text{GeV}}{\left(0.200\ \text{GeV/c}^2\right)c^2} = \underline{5\times 10^{10}}$$

(b) Divide the number of particles by the area they hit:

$$\text{particles/m}^2 = \frac{5\times 10^{10}\ \text{particles}}{(1000\ \text{m})^2} = \underline{5\times 10^4\ \text{particles/m}^2}$$

49. ***Integrated Concepts*** *Suppose you are designing a proton decay experiment and you can detect 50 percent of the proton decays in a tank of water. (a) How many kilograms of water would you need to see one decay per month, assuming a lifetime of* $10^{31}\ \text{y}$*? (b) How many cubic meters of water is this? (c) If the actual lifetime is* $10^{33}\ \text{y}$, *how long would you have to wait on an average to see a single proton decay?*

Solution (a) On average, one proton decays every $10^{31}\ \text{y} = 12\times 10^{31}\ \text{months}$. So for one decay every month, you would need:

$$N\left(\frac{1}{12\times10^{31}\ \text{months/decay}}\right)=\frac{1\ \text{decay}}{\text{month}}\Rightarrow N=12\times10^{31}\ \text{protons}$$

Since you detect only 50% of the actual decays, you need twice this number of protons to observe one decay per month, or $N=24\times10^{31}\ \text{protons}$. Now, we know that one H_2O molecule has 10 protons (1 from each hydrogen plus 8 from the oxygen), so we need $24\times10^{30}\ H_2O$. Finally, since we know how many molecules we need, and we know the molar mass of water, we can determine the number of kilograms of water we need.

$$\left(24\times10^{30}\ \text{molecules}\right)\left(\frac{1\ \text{mole}}{6.02\times10^{23}\ \text{molecules}}\right)\left(\frac{0.018\ \text{kg}}{\text{mole}}\right)=\underline{7.2\times10^{5}\ \text{kg of water}}$$

(b) Now, we know the density of water, $\rho=1000\ \text{kg/m}^3$, so we can determine the volume of water we need: $V=m\rho=\left(7.2\times10^5\ \text{kg}\right)\left(\frac{1\ \text{m}^3}{1000\ \text{kg}}\right)=\underline{7.2\times10^2\ \text{m}^3}$

(c) If we had $7.2\times10^2\ \text{m}^3$ of water, and the actual decay rate was $10^{33}\ \text{y}$, rather than $10^{31}\ \text{y}$, a decay would occur 100 times less often, and we would have to wait on average 100 months to see a decay.

CHAPTER 34: FRONTIERS OF PHYSICS

34.1 COSMOLOGY AND PARTICLE PHYSICS

1. *Find the approximate mass of the luminous matter in the Milky Way galaxy, given it has approximately* 10^{11} *stars of average mass 1.5 times that of our Sun.*

Solution The approximate mass of the luminous matter in the Milky Way galaxy can be found by multiplying the number of stars times 1.5 times the mass of our Sun:

$$M = (10^{11})(1.5)m_S = (10^{11})(1.5)(1.99\times10^{30}\text{ kg}) = \underline{3\times10^{41}\text{ kg}}$$

7. *(a) What is the approximate velocity relative to us of a galaxy near the edge of the known universe, some 10 Gly away? (b) What fraction of the speed of light is this? Note that we have observed galaxies moving away from us at greater than* $0.9c$.

Solution (a) Using $v = H_0 d$ and the Hubble constant, we can calculate the approximate velocity of the near edge of the known universe:

$$v = H_0 d = (20\text{ km/s}\cdot\text{Mly})(10\times10^3\text{ Mly}) = \underline{2.0\times10^5\text{ km/s}}$$

(b) To calculate the fraction of the speed of light, divide this velocity by the speed of light: $\frac{v}{c} = \frac{(2.0\times10^5\text{ km/s})(10^3\text{ m/km})}{3.00\times10^8\text{ m/s}} = 0.67$, so that $\underline{v = 0.67c}$

11. *Andromeda galaxy is the closest large galaxy and is visible to the naked eye. Estimate its brightness relative to the Sun, assuming it has luminosity* 10^{12} *times that of the Sun and lies 2 Mly away.*

Solution The relative brightness of a star is going to be proportional to the ratio of surface areas times the luminosity, so:

$$\text{Relative Brightness} = (\text{luminosity})\frac{4\pi r^2}{4\pi R_{\text{Andromeda}}^2} = \left(10^{12}\right)\left(\frac{r_{\text{Sun}}}{R_{\text{Andromeda}}}\right)^2.$$

From Appendix C, we know the average distance to the sun is $1.496\times10^{11}\ \text{m}$, and we are told the average distance to Andromeda, so:

$$\text{Relative Brightness} = \frac{\left(10^{12}\right)\left(1.496\times10^{11}\ \text{m}\right)^2}{\left[\left(2\times10^{6}\ \text{ly}\right)\left(9.46\times10^{15}\ \text{m/ly}\right)\right]^2} = \underline{6\times10^{-11}}.$$

Note: this is an overestimate since some of the light from Andromeda is blocked by its own gas and dust clouds.

15. *(a) What Hubble constant corresponds to an approximate age of the universe of 10^{10} y? To get an approximate value, assume the expansion rate is constant and calculate the speed at which two galaxies must move apart to be separated by 1 Mly (present average galactic separation) in a time of 10^{10} y. (b) Similarly, what Hubble constant corresponds to a universe approximately 2×10^{10}-y old?*

Solution (a) Since the Hubble constant has units of $\text{km/s}\cdot\text{Mly}$, we can calculate its value by considering the age of the universe and the average galactic separation. If the universe is 10^{10} years old, then it will take 10^{10} years for the galaxies to travel 1 Mly. Now, to determine the value for the Hubble constant, we just need to determine the average velocity of the galaxies from the equation $d = v\times t$:

$$v = \frac{d}{t},\ \text{so that}\ v = \frac{1\ \text{Mly}}{10^{10}\ \text{y}} = \frac{1\times10^{6}\ \text{ly}}{10^{10}\ \text{y}}\times\frac{9.46\times10^{12}\ \text{km}}{1\ \text{ly}}\times\frac{1\ \text{y}}{3.156\times10^{7}\ \text{s}} = 30\ \text{km/s}.$$

$$\text{Thus, } H_0 = \frac{30\ \text{km/s}}{1\ \text{Mly}} = \underline{30\ \text{km/s}\cdot\text{Mly}}$$

(b) Now, the time is 2×10^{10} years, so the velocity becomes:

$$v=\frac{1\,\text{Mly}}{2\times10^{10}\ \text{y}}=\frac{1\times10^{6}\ \text{ly}}{2\times10^{10}\ \text{y}}\times\frac{9.46\times10^{12}\ \text{km}}{1\,\text{ly}}\times\frac{1\,\text{y}}{3.156\times10^{7}\ \text{s}}=15\ \text{km/s}.$$

Thus, the Hubble constant would be approximately $H_0=\frac{15\ \text{km/s}}{1\,\text{Mly}}=15\,\text{km/s}\cdot\text{Mly}$

16. *Show that the velocity of a star orbiting its galaxy in a circular orbit is inversely proportional to the square root of its orbital radius, assuming the mass of the stars inside its orbit acts like a single mass at the center of the galaxy. You may use an equation from a previous chapter to support your conclusion, but you must justify its use and define all terms used.*

Solution A star orbiting its galaxy in a circular orbit feels the gravitational force acting toward the center, which is the centripetal force (keeping the star orbiting in a circle). So, from $F=G\frac{mM}{r^2}$, we get an expression for the gravitational force acting on the star, and from $F_\text{c}=m\frac{v^2}{r}$, we get an expression for the centripetal force keeping the star orbiting in a circle. Setting the two forces equal gives: $F=\frac{mv^2}{r}=\frac{GMm}{r^2}$, where m is the mass of the star, M is the mass of the galaxy (assumed to be concentrated at the center of the rotation), G is the gravitational constant, v is the velocity of the star, and r is the orbital radius. Solving the equation for the velocity gives: $v=\sqrt{\frac{GM}{r}}$ so that the velocity of a star orbiting its galaxy in a circular orbit is indeed inversely proportional to the square root of its orbital radius.